KEMENTERIAN PENDIDIKAN MALAYSIA
Jabatan Pendidikan Negeri Sabah

MODULE

ADDITIONAL MATHEMATICS

FORM 5
2021

(ENGLISH VERSION)

JABATAN PENDIDIKAN NEGERI SABAH

PREFACE

Understanding the concept and doing a lot of practice on the concepts learned is a contributing factor to student success in the SPM examination. Most students who are weak in mastering the concept are due to lack of practice. A compact and brief note (One Page Note) that is included in this module is expected to help students to master the concept of the topic. Doing intensive exercises for each topic is also expected to help teachers and students master the topic.

Therefore, this module is hoped to help teachers and students during teaching and learning and during revision exercises before the SPM examination.

MODULE OBJECTIVE

1. Make it easier for students to remember important concepts in the form of more compact graphics.

2. Help students answer revision practice questions before the SPM examination

3. Help teachers overcome the problems of students who do not have any reference sources.

Sincerely from:

Lee Chiong Tee
Coordinator and Head Panels
Additional Mathematics Module 2021

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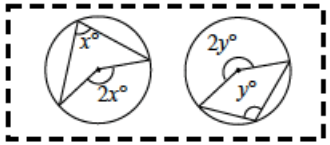
CIRCULAR MEASURE

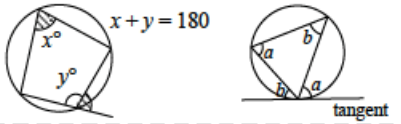
- ONE PAGE NOTE (OPN)
- WORKSHEET

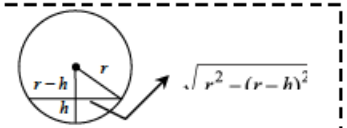
Encik Peter Wong Yung Ming

ONE PAGE NOTES

" CIRCULAR MEASURE "



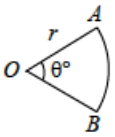




Recognise Cirlce

Circumference = $2\pi r = \pi d$

Area = πr^2



Arc length of AB, $S_{AB} = \frac{\theta}{360} \times 2\pi r$

Area of sector OAB, $A_{OAB} = \frac{\theta}{360} \times \pi r^2$

major arc ← → minor arc

major sector ← → minor sector

major segment ← → minor segment

Unit Coversion

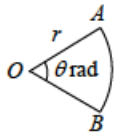
degrees $\xrightarrow{\times \frac{\pi}{180}}$ radians

radians $\xrightarrow{\times \frac{180}{\pi}}$ degrees

《 180° = π 》

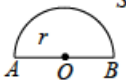
30° = $\frac{\pi}{6}$ rad		90° = $\frac{\pi}{2}$ rad	
45° = $\frac{\pi}{4}$ rad		270° = $\frac{3\pi}{2}$ rad	
60° = $\frac{\pi}{3}$ rad		360° = 2π rad	

Arc Length & Area of Sector



$S_{AB} = \theta r$

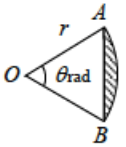
$A_{OAB} = \frac{1}{2}\theta r^2$



$S_{AB} = \pi r = \frac{1}{2}\pi d$

$A_{OAB} = \frac{1}{2}\pi r^2$

Length of Chord, Area Δ & Area of Segment



Chord AB = $2r \sin\left(\frac{\theta}{2}\right)$

Area $\Delta OAB = \frac{1}{2} r^2 \sin\theta$

Area of segment = $\frac{1}{2} r^2 (\theta - \sin\theta)$

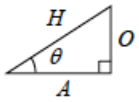
Pythagoras' Theorems & Trigonometric Ratio → Find : θ @ radius @ length of side

$H = \sqrt{O^2 + A^2}$

$O = \sqrt{H^2 - A^2}$

$A = \sqrt{H^2 - O^2}$

3	4	5	5	12	13
6	8	10	8	15	17
9	12	15	7	24	25



area Δ = $\frac{1}{2} \times O \times A$

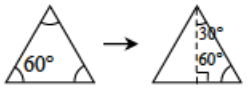
$\sin\theta = \frac{O}{H}$

$\cos\theta = \frac{A}{H}$

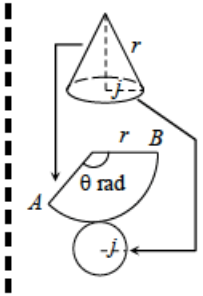
$\tan\theta = \frac{O}{A}$

Isosceles Triangle

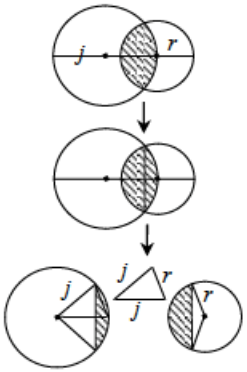
Equilateral Triangle

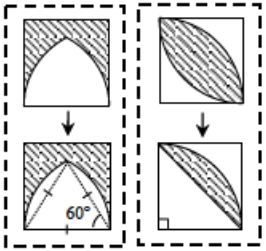


Cone & Layout

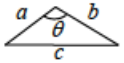


$S_{AB} = \theta r = 2\pi j$





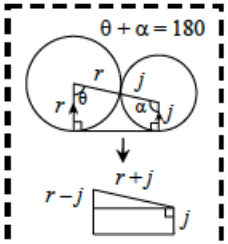
Solution of Triangle

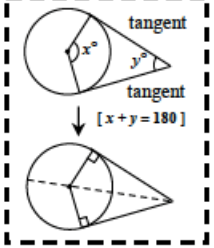


$\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$

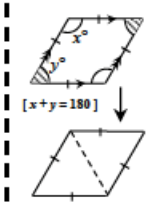
area Δ = $\frac{1}{2} ab \sin\theta$

$c^2 = a^2 + b^2 - 2ab\cos\theta$





Rhombus



"Heron's formula" : area Δ = $\sqrt{s(s-a)(s-b)(s-c)}$ $s = \frac{a+b+c}{2}$ = semi perimeter

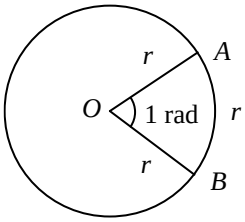
WORKSHEET
TOPIC 1 : CIRCULAR MEASURE
[1 question → 3 – 4 marks]

==========

1.1 Radian
1.1.1 Relate angle measurement in radian and degree.

==========

1 Complete each of the following :



In general, a circle with centre O, and radius r units :

- If $\angle OAB = 1$ radian $\rightarrow S_{AB} =$ ☐
- If $\angle OAB = 2$ radians $\rightarrow S_{AB} =$ ☐
- If $\angle OAB = 3$ radians $\rightarrow S_{AB} =$ ☐

⋮

- If $\angle OAB = \pi$ radians $\rightarrow S_{AB} =$ ☐

⋮

- If $\angle OAB = 2\pi$ radians $\rightarrow S_{AB} =$ ☐

Conclusion :

2π radians $=$ ☐ °

↓

π radians $=$ ☐ °

☐

$\times \frac{\pi}{180}$

degrees ⇄ radians

$\times \frac{180}{\pi}$

☐

MIND think :

☐ **radian** is the measure of an angle substended at the centre of a circle by an arc in which its leng is same as the radius of the circle.

2 Convert each of the special angles in terms of π radians.

(a) 30° =	(b) 45° =	(c) 60° =	(d) 90° =
(e) 120° =	(f) 180° =	(g) 270° =	(h) 360° =

3 Convert each of the following angles into radians. Give your answer correct to four significant figures. [Use $\pi = 3.142$] $\rightarrow \times \frac{\pi}{180}$

(a) 36° =	(b) 75° =	(c) 105.7° =	(d) 224° 36′ =

4 Convert each of the following angles into degrees. Give your answer correct to two decimal places. [Use $\pi = 3.142$] $\rightarrow \times \frac{180}{\pi}$

(a) 3.24 rad =	(b) $\frac{2}{3}$ rad =	(c) $1\frac{1}{4}$ rad =	(d) $\frac{14}{9}\pi$ rad =

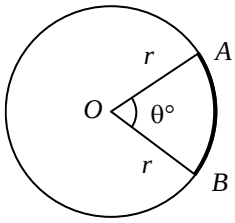
==========

1.2 Arc Length of a Circle

1.2.1 Determine : (i) arc length, (ii) radius, and (iii) angle subtended at the centre of a circle.

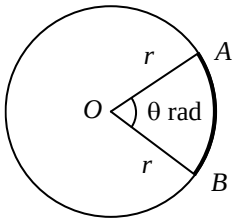
==========

5 Complete each of the following :



$\frac{S_{AB}}{\theta°} = \frac{2\pi r}{360°}$

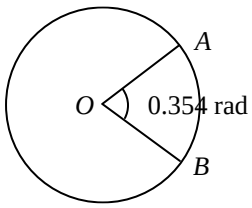
S_{AB} =

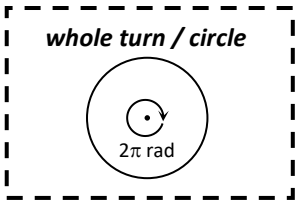


$\frac{S_{AB}}{\theta \text{ rad}} = \frac{2\pi r}{2\pi \text{ rad}}$

S_{AB} =

6 The diagram shows a circle with centre *O*.





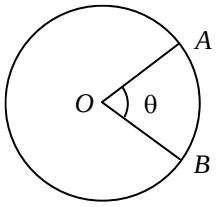
Given the length of the major arc *AB* is 45.51 cm. Find the length, in cm, of the radius.[Use $\pi = 3.142$]

(*Ans*: 7.675)

[3 marks] [***2004, No.19***]

Answer :

7 The diagram shows a circle with centre O.



The length of the minor arc AB is 16 cm and the angle of the major sector AOB is 290°. Using $\pi = 3.142$, find

(a) the value of θ, in radians, (Give your answer correct to four significant figures) (*Ans* : 1.222)

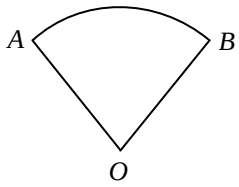
(b) the length, in cm, of the radius of the circle. (*Ans* : 13.093)

[3 marks] [***2005, No.18***]

Answer :

(a) (b)

8 In the diagram, the length of the arc AB is 1.5 times its radius OA.

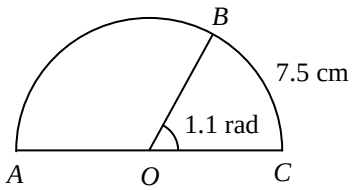


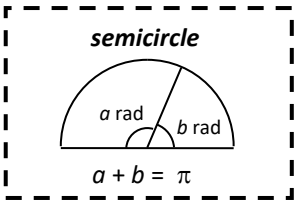
Find $\angle AOB$, in degrees. Give your answer correct to two decimal places. [Use $\pi = 3.142$]
(*Ans* : 85.93)

[2 marks] [***Forecast***]

Answer :

9 In the diagram, ABC is a semicircle with centre O.



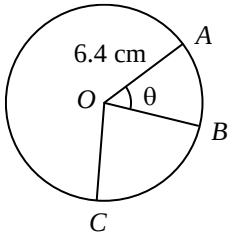


Given that the length of the arc BC is 7.5 cm and $\angle BOC = 1.1$ rad. Find the length of the arc AB.
[Use $\pi = 3.142$] (*Ans* : 13.92)

[3 marks] [***Forecast***]

Answer :

10 The diagram shows a circle with centre *O*.



The length of minor arc *AB* is 4.8 cm and the angle of the minor sector *BOC* is 1 rad. Using $\pi = 3.142$, find

(a) the value of θ, in radians, (*Ans* : 0.75)

(b) the length, in cm, of the major arc *AC*. (*Ans* : 29.02)

[4 marks] [***Forecast***]

Answer :

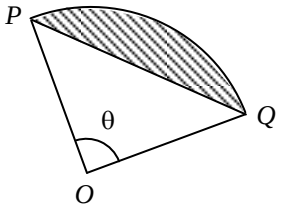
(a) (b)

==========

1.2.2 Determine perimeter of segment of a circle.

==========

11 The diagram shows a sector with centre *O*, and a radius of 2.75 cm.



Given that the arc length of *PQ* is 4.62 cm. Use $\pi = 3.142$, find

(a) the angle θ, in degrees, (*Ans* : 96.24)

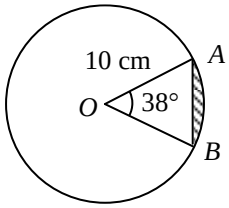
(b) the perimeter, in cm, of the shaded segment. (*Ans* : 8.715)

[4 marks] [***Forecast***]

Answer :

(a) (b)

12 The diagram shows a circle with centre *O*.

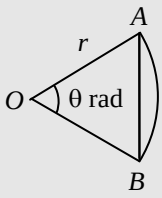


Find the perimeter, in cm, of the shaded segment. [Use $\pi = 3.142$] (*Ans* : 13.144)

[4 marks] [***Forecast***]

Answer :

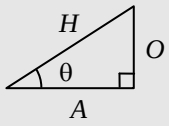
MIND think :



$chord\ AB = 2r \sin\left(\frac{\theta}{2}\right)$

$AB^2 = r^2 + r^2 - 2r^2 \cos\theta$

$\cos\theta = \frac{OA^2 + OB^2 - AB^2}{2(OA)(OB)}$



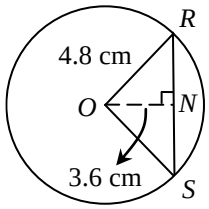
area $\Delta = \frac{1}{2} \times O \times A$

$\sin\theta = \frac{O}{H}$ $\quad H = \sqrt{O^2 + A^2}$

$\cos\theta = \frac{A}{H}$ $\quad O = \sqrt{H^2 - A^2}$

$\tan\theta = \frac{O}{A}$ $\quad A = \sqrt{H^2 - O^2}$

13 The diagram shows a circle with centre *O* and of radius 4.8 cm.



N is the midpoint of chord *RS* and *ON* = 3.6 cm. Using $\pi = 3.142$, find

(a) $\angle ROS$, in radians, (*Ans* : 1.446)

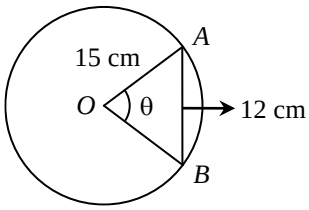
(b) the perimeter, in cm, of the minor segment *RS*. (*Ans* : 13.29)

[4 marks] [***Forecast***]

Answer :

(a) (b)

14 The diagram shows a circle of radius 15 cm, with a chord AB of 12 cm.



Using $\pi = 3.142$, find

(a) the value of θ, in radians, (*Ans* : 0.8232)

(b) the perimeter, in cm, of the minor segment AB. (*Ans* : 24.348)

[4 marks] [***Forecast***]

Answer :

(a) (b)

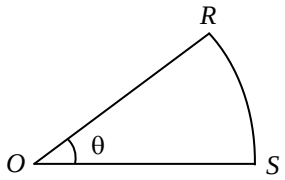
==========

1.2.3 Solve problems involving arc length.

==========

⇒ ***problems solving 1***

15 The diagram shows a sector ROS with centre O.

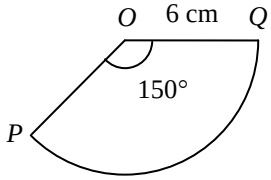


The length of the arc RS is 7.24 cm and the perimeter of the sector ROS is 25 cm. Find the value of θ, in rad. (*Ans* : 0.8153)

[3 marks] [***2003, No.19***]

Answer :

16 The diagram shows the sector *OPQ* with centre *O*.



Find [Use $\pi = 3.142$]

(a) $\angle POQ$, in terms of π radian, (*Ans* : $\frac{5}{6}\pi$)

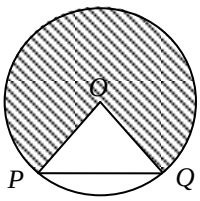
(b) the perimeter, in cm, sector *OPQ*. (*Ans* : 27.71)

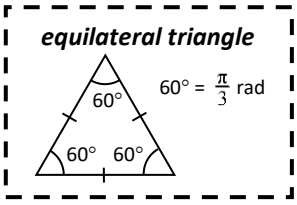
[4 marks] [***2013, No.17***]

Answer :

(a) (b)

17 The diagram shows a circle with centre *O* and of radius $OP = PQ = 3\frac{1}{2}$ cm.



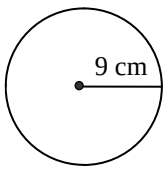


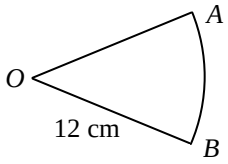
Find the perimeter, in cm, of the shaded region. [Use $\pi = 3.142$] (*Ans* : 25.328)

[3 marks] [***Forecast***]

Answer :

18 The diagram shows a piece of wire in shape of a circle with radius 9 cm is bent to form a sector *AOB* with centre *O* and of radius 12 cm.



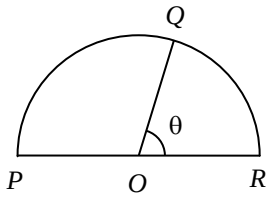


Find the angle of the sector *AOB*, in radians. [Use $\pi = 3.142$] (*Ans* : 2.713)

[3 marks] [***Forecast***]

Answer :

19 The diagram shows a semicircle with centre *O* and radius 8 cm.

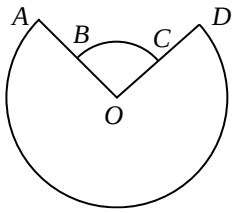


Given the length of arc *PQ* is equal to perimeter of minor sector *OQR*. Find the value of θ, in radians.
[Use $\pi = 3.142$] (*Ans* : 0.571)
[3 marks] [***Forecast***]

Answer :

⇒ ***problems solving 2***

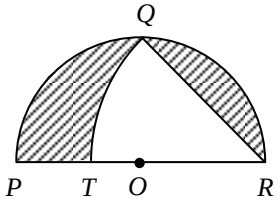
20 The diagram shows two sectors *AOD* and *BOC* of two concentric circles with centre *O*.



The angle subtended at the centre *O* by the major arc *AD* is 7α radians and the perimeter of the whole diagram is 50 cm. Given *OB* = *r* cm, *OA* = 2*OB* and $\angle BOC = 2\alpha$, express *r* in terms of α.
(*Ans* : $\frac{25}{8\alpha + 1}$)
[3 marks] [*2017, No.16*]

Answer :

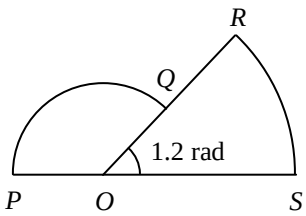
21 The diagram shows a semicircle *PQR* with centre *O* and sector *RTQ* with centre *R*.



Given *OR* = 8 cm, *TR* = 12 cm and $\angle TRQ = 1.134$ radians. Find the perimeter, in cm, of the shaded region. [Use $\pi = 3.142$] (*Ans* : 54.744)
[3 marks] [***Forecast***]

Answer :

22 The diagram shows two sectors *POQ* and *ROS* with centre *O*. *POS* and *OQR* are straight lines.

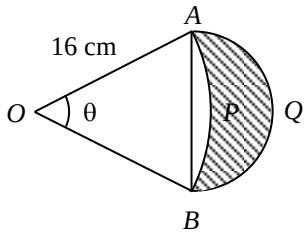


Given $OP = 5$ cm, $OS = 2OP$, and $\angle ROS = 1.2$ rad. Find the perimeter, in cm, of the whole diagram.
[Use $\pi = 3.142$] (*Ans* : 41.71)

[3 marks] [***Forecast***]

Answer :

23 The diagram shows an arc *APB*, with centre *O* and of radius 16 cm and a semicircle *AQB*, with diameter *AB*.



Given that the triangle *AOB* is an equilateral triangle and $\angle AOB$ is θ. Using $\pi = 3.142$, find

(a) the value of θ, in radians, (*Ans* : 1.047)

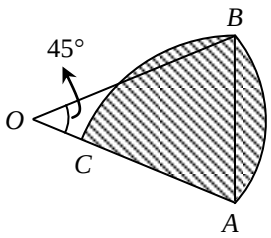
(b) the perimeter, in cm, of the shaded region. (*Ans* : 41.89)

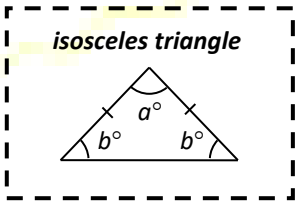
[4 marks] [***Forecast***]

Answer :

(a) (b)

24 The diagram shows a sector *AOB* wirh centre *O* and radius 10 cm, and a sector *CBA* with centre *A*.





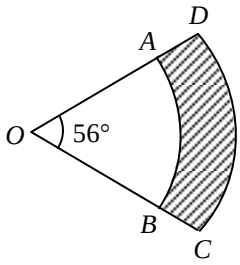
Given that $\angle BOA = 45°$, find the perimeter, in cm, of the shaded region. [Use $\pi = 3.142$]
(*Ans* : 24.52)

[4 marks] [***Forecast***]

Answer :

⇒ ***problems solving 3***

25 The diagram shows two sectors *AOB* and *COD* with centre *O*.

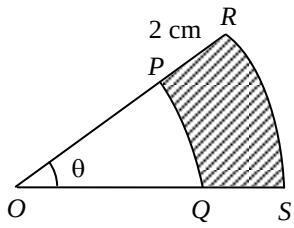


Given the radius *OA* and *OC* are 11 cm and 15 cm respectively. If $\angle AOB$ is 56°, find the perimeter, in cm, of the shaded region. [Use $\pi = 3.142$]
(*Ans* : 33.415)

[3 marks] [***Forecast***]

Answer :

26 The diagram shows two sectors POQ and ROS with centre O.

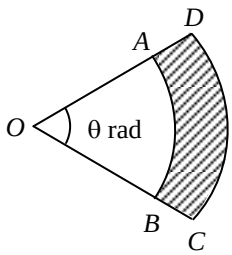


Given $\angle POQ = 0.8$ rad and $OP = 3PR$. Find the perimeter, in cm, of the shaded region.
(*Ans* : 15.2)

[3 marks] [***Forecast***]

Answer :

27 The diagram shows two arcs AB and CD with a common centre O.



Given that the length of the arc AB is 2 times its radius OB, and $OC = 15$cm. Find

(a) the angle of θ, in radians, (*Ans* : 2)

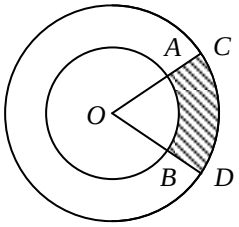
(b) the perimeter, in cm, of the shaded region $ABCD$. (*Ans* : 60)

[4 marks] [***Forecast***]

Answer :

(a) (b)

28 In the diagram, AB and CD are two arcs of two circles that have the same centre O.



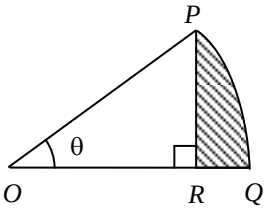
Given that the length of the arcs AB and CD are 13.1 cm and 23.6 cm respectively. If the radius of the small circle is 6 cm, find the perimeter, in cm, of the shaded region $ABCD$. (*Ans* : 46.32)

[4 marks] [***Forecast***]

Answer :

⇒ ***problems solving 4***

29 The diagram shows a sector POQ of a circle with centre O.



It is given that OR = 8 cm and OP = 10 cm. Using π = 3.142, find

(a) the value of θ, in radians, (*Ans* : 0.6436)

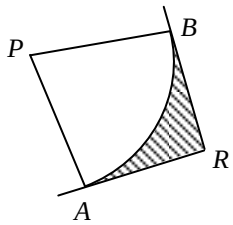
(b) the perimeter, in cm, of the shaded region. (*Ans* : 14.43)

[4 marks] [***2011, No.18***]

Answer :

(a) (b)

30 In the diagram, points A and B lie on a circumference of a circle with centre P and a radius of 6 cm.

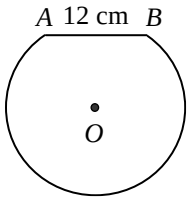


Tangen to the circle at point A and point B meet at R and $\angle APB$ = 1.4 radians. Find the perimeter, in cm, of the shaded region. [Use π = 3.142] (*Ans* : 18.51)

[4 marks] [***Forecast***]

Answer :

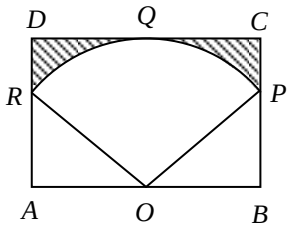
31 The diagram shows an incomplete circle, in which the minor segment is cut from a circle of radius 10 cm.



Given that the chord AB = 12 cm. Find the perimeter, in cm, of the whole diagram.
[Use π = 3.142] (*Ans* : 61.96)
[4 marks] [***Forecast***]

Answer :

32 The diagram shows a sector $OPQR$ with centre O inscribed in a rectangle $ABCD$.



Given AB = 20 cm and BC = 15 cm. Using π = 3.142, calculate

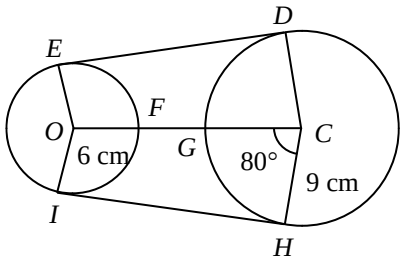
(a) $\angle POR$, in radians, (*Ans* : 1.460)

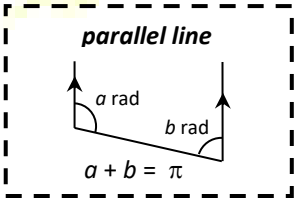
(b) the perimeter, in cm, of the shaded region. (*Ans* : 49.54)
[4 marks] [***Forecast***]

Answer :

(a) (b)

33 In the diagram, *DE* and *HI* are common tangents to two circles with centres *O* and *C* respectively.





OFGC is straight line. Using $\pi = 3.142$, calculate

(a) the length, in cm, of *DE*, (*Ans* : 17.01)

(b) the perimeter, in cm, of the whole diagram. (*Ans* : 82.197)

[4 marks] [***Forecast***]

Answer :

(a) (b)

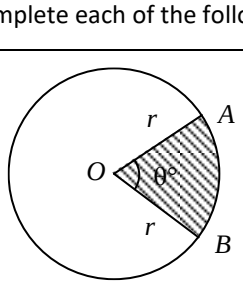
==========

1.3 Area of Sector of a Circle

1.3.1 Determine : (i) area of sector, (ii) radius, and (iii) angle subtended at the centre of a circle.

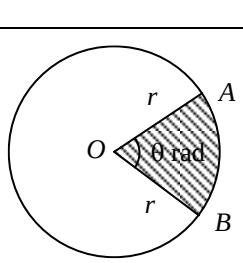
==========

34 Complete each of the following :



$\frac{A_{OAB}}{\theta^\circ} = \frac{\pi r^2}{360^\circ}$

$A_{OAB} =$

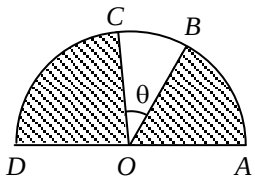


$\frac{A_{OAB}}{\theta \text{ rad}} = \frac{\pi r^2}{2\pi \text{ rad}}$

$A_{OAB} =$

⇒ ***solve 1***

35 The diagram shows a semicircle with centre *O* and of radius 6 cm.

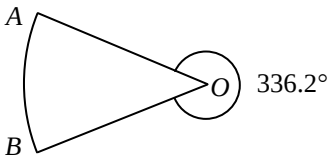


Given that $\theta = 0.7$ radian, find the area, in cm^2, of shaded region. [Use $\pi = 3.142$]
(*Ans* : 43.96)

[2marks] [***Forecast***]

Answer :

36 The diagram shows a sector *AOB* with centre *O*.

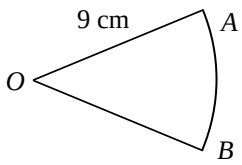


Given that the area of the sector is 20 cm^2, find the length, in cm, of the radius.
[Use $\pi = 3.142$] (*Ans* : 9.812)

[2 marks] [***Forecast***]

Answer :

37 The diagram shows a sector with centre *O*.



If the area of the sector *AOB* is 36 cm^2, find the $\angle AOB$, in radians. (*Ans* : $\frac{8}{9}$)

[2marks] [***Forecast***]

Answer :

⇒ ***solve 2***

38 A piece of wire, 20 cm in length is used to form a sector of a circle with a radius of 6 cm. Find

(a) the angle of the sector, in radians, (*Ans* : 1.333)

(b) the area, in cm^2, of the sector. (*Ans* : 23.99)

[4 marks] [***Forecast***]

Answer :

(a) (b)

39 A piece of wire is bent to form a sector of a circle with centre *O* and of radius 6 cm. If the area of the sector is 18 cm^2, find the length, in cm, of the wire. (*Ans* : 18)

[4 marks] [*Forecast*]

Answer :

40 *AB* is the arc of a circle with centre *O*. If the length of the arc *AB* is 15 cm and the area of the sector *AOB* is 90 cm^2, find

(a) the radius, in cm, of the sector, (*Ans* : 12)

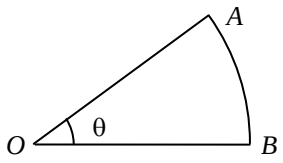
(b) $\angle AOB$, in radians. (*Ans* : 1.25)

[4 marks] [***Forecast***]

Answer :

(a) (b)

41 The diagram shows a sector *AOB* with centre *O*.



If the area of the sector *AOB* is 1200 cm^2, and the ratio of the arc *AB* to the radius is 2 : 3, using π = 3.142, find

(a) the value of θ, in degrees and minutes, (*Ans* : 38°12′)

(b) the radius, in cm, of the sector *AOB*. (*Ans* : 60)

[4 marks] [***Forecast***]

Answer :

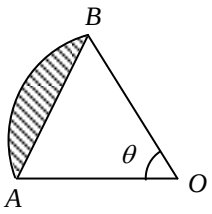
(a) (b)

==========

1.3.2 Determine the area of segment of a circle.

==========

42 The diagram shows a sector *OAB* with centre *O* and radius 8 cm.



Given *OA* = *OB* = *AB*, find [Use π = 3.142]

(a) the value of θ, in radians, (*Ans* : 1.047)

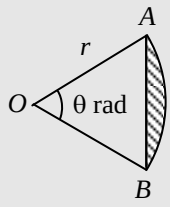
(b) the area of the shaded region. (*Ans* : 5.791)

[4 marks] [*2015, No.18*]

Answer :

(a) (b)

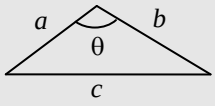
MIND think :



$Area_{\Delta OAB} = \frac{1}{2} r^2 \sin\theta$

$Area_{segment} = \frac{1}{2} r^2 (\theta - \sin\theta)$

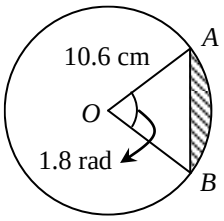
$c^2 = a^2 + b^2 - 2ab\cos\theta$ $\quad area\ \Delta = \frac{1}{2} ab \sin\theta$



$\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$

$area\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$, $s = \frac{a+b+c}{2}$

43 The diagram shows a circle with centre *O*.

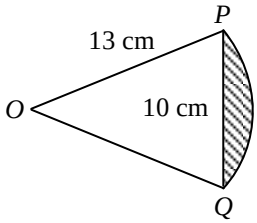


Find the area, in cm^2, of the shaded segment. (*Ans* : 46.414)

[2 marks] [***Forecast***]

Answer :

44 The diagram shows a sector *POQ* with centre *O* and of radius 13 cm.

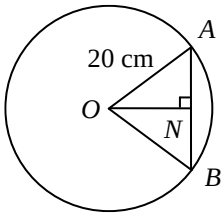


Given *PQ* = 10 cm. Using π = 3.142, find the area, in cm^2, of the shaded segment.
(*Ans* : 6.729 or 6.722)

[4 marks] [***Forecast***]

Answer :

45 The diagram shows a circle with centre O and of radius 20 cm.



N is the midpoint of chord AB and ON = 16 cm. Use $\pi = 3.142$, find

(a) $\angle AOB$, in radians, (*Ans* : 1.287)

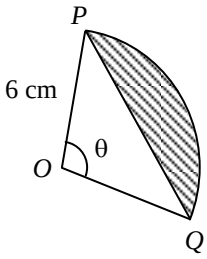
(b) the area, in cm^2, of the minor segment. (*Ans* : 65.40)

[4 marks] [***Forecast***]

Answer :

(a) (b)

46 The diagram shows a sector POQ with centre O.



Given that the length of arc PQ is 10 cm. Using $\pi = 3.142$, find

(a) the value of θ, in radians, (*Ans* : $\frac{5}{3}$)

(b) the area, in cm^2, of the shaded region. (*Ans* : 12.08)

[4 marks] [***Forecast***]

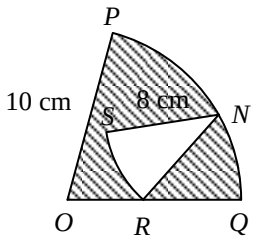
Answer :

(a) (b)

1.3.2 Solve problems involving areas of sectors.

⇒ ***problems solving 1***

47 The diagram shows sector OPQ of a circle with centre O, and sectors NRS of a circle with centre N.

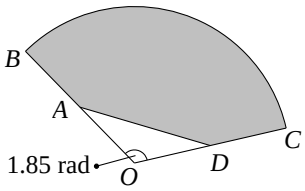


Given $\angle POQ = 1.5$ radians and $\angle RNS = 0.5$ radians, find the area, in cm^2, of the shaded region.
(*Ans* : 59)

[3 marks] [*2010, No.17*]

Answer :

48 The diagram shows a sector BOC of a circle with centre O.



It is given that $AD = 8$ cm and $BA = AO = OD = DC = 5$ cm. Using $\pi = 3.142$, find

(a) the length, in cm, of the arc BC, (*Ans* : 18.5)

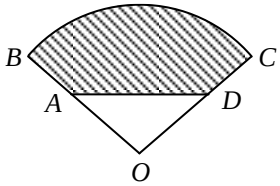
(b) the area, in cm^2, of the shaded region. (*Ans* : 80.48)

[4 marks] [*2007, No.18*]

Answer :

(a) (b)

49 The diagram shows a sector BOC of a circle with centre O.



It is given that $\angle BOC = 1.42$ radians, and $OA = AB = OD = DC = 5$ cm. Find

(a) the length, in cm, of arc BC, (*Ans* : 14.2)

(b) the area, in cm^2, of the shaded region. (*Ans* : 58.64)

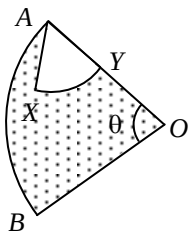
[4 marks] [*2009, No.12*]

Answer :

(a) (b)

⇒ ***problems solving 2***

50 The diagram shows sector OAB with centre O and sector AXY with centre A.



Given that $OB = 10$ cm, $AY = 4$ cm, $\angle XAY = 1.1$ radians and the length of arc $AB = 7$ cm, calculate

(a) the value of θ, in radians, (*Ans* : 0.7)

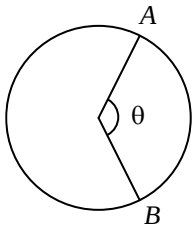
(b) the area, in cm^2, of the shaded region. (*Ans* : 26.2)

[4 marks] [***2006, No.16***]

Answer :

(a) (b)

51 The diagram shows a circle with centre O and radius 8 cm.



Given the length of the minor arc AB is 16 cm. [Use $\pi = 3.142$]

(a) State the value of θ, in radians. (*Ans* : 2)

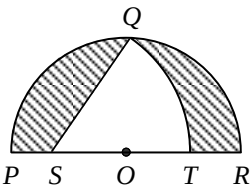
(b) Find the area of the major sector OAB, in cm^2, correct to four significant figures. (*Ans* : 137.1)

[4 marks] [*2016, No.19*]

Answer :

(a) (b)

52 The diagram shows a semicircle PQR with centre O, and a sector QST with centre S.



If $\angle QST = 0.9$ rad, $OP = 6$ cm, and the length of the arc $QT = 6.3$ cm, Using $\pi = 3.142$, find

(a) the length, in cm, of ST, (*Ans* : 7)

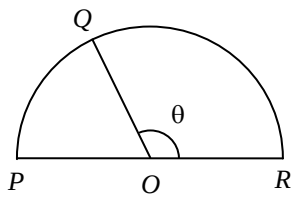
(b) the area, in cm^2, of the shaded region. (*Ans* : 34.51)

[4 marks] [***Forecast***]

Answer :

(a) (b)

53 In the diagram, POR is the diameter of a circle with centre O.



Given that the length of the arc RQ is equal to the perimeter of the sector POQ, and the radius of the circle is 6 cm. Using $\pi = 3.142$, find

(a) the angle θ, in radians, (*Ans* : 2.571)

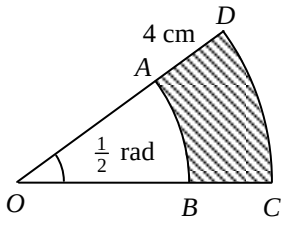
(b) the area, in cm^2, of the sector ROQ. (*Ans* : 46.28)

[4 marks] [***Forecast***]

Answer :

(a) (b)

54 The diagram shows two sectors AOB and COD with centre O.



Given $BC = AD = 4$ cm and $\angle AOB = \frac{1}{2}$ radian. If the perimeter of $ABCD$ is 16 cm, find

(a) the length, in cm, of OB, (*Ans* : 6)

(b) the area, in cm^2, of the shaded region. (*Ans* : 16)

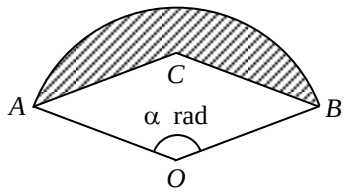
[4 marks] [***Forecast***]

Answer :

(a) (b)

⇒ ***problems solving 3***

55 The diagram shows a rhombus *OABC* inscribed in sector *AOB* with centre *O* and radius *r* cm.



Given the area of sector *AOB* is 18 cm^2, express

(a) α in terms of *r*, (*Ans* : $\frac{36}{r^2}$)

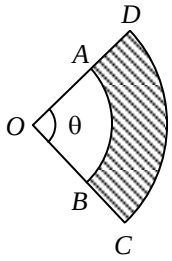
(b) the perimeter, in cm, of the shaded region in terms of *r*. (*Ans* : $\frac{2r^2+36}{r}$)

[3 marks] [***2019, No.12***]

Answer :

(a) (b)

56 The diagram shows a sectors *OAB* and *ODC* with centre *O*.



It is given that *OA* = 4 cm, the ratio of *OA* : *OD* = 2 : 3 and the area of the shaded region is 11.25 cm^2. Find

(a) the length, in cm, of *OD*, (*Ans* : 6)

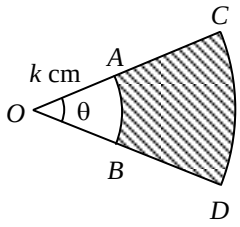
(b) θ, in radians. (*Ans* : 1.125)

[4 marks] [*2012, No.18*]

Answer :

(a) (b)

57 The diagram shows sector *OAB* and sector *OCD*, with centre *O*.



Given that $\theta = 0.8$ rad, $OA : OC = 1 : 3$, and the area of sector OAB is 40 cm^2. Using $\pi = 3.142$, find

(a) the value of k, (*Ans* : 10)

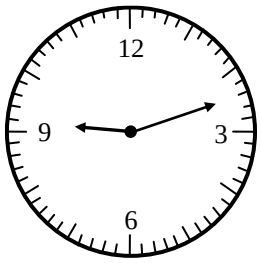
(b) the perimeter, in cm, of the shaded region. (*Ans* : 72)

[4 marks] [***Forecast***]

Answer :

(a) (b)

58 The diagram shows a wall clock showing the time 9.12 am



$\angle$ hour hand = $\frac{1}{12}$ $\angle$ minute hand

$\angle$ minute hand = $\frac{1}{60}$ $\angle$ second hand

Given that the lengths of hour hand and minute hand are 2 cm and 4 cm respectively. [Use $\pi = 3.142$]

(a) Find the time, when the area swept through by the minute hand is 30.1632 cm^2. (*Ans* : 9.48am)

(b) Hence, find the area swept through by the hour hand, in cm^2. (*Ans* : 0.6284)

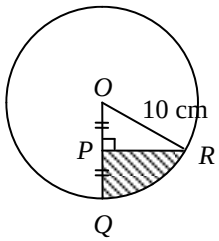
[4 marks] [***Forecast***]

Answer :

(a) (b)

⇒ ***problems solving 4***

59 The diagram shows a circle with centre O and radius 10 cm.



Given that P, Q and R are points such that $OP = PQ$ and $\angle OPR = 90°$, using $\pi = 3.142$, find

(a) $\angle QOR$, in radians, (*Ans* : 1.047)

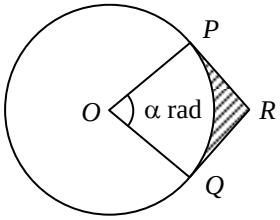
(b) the area, in cm^2, of the shaded region. (*Ans* : 30.7)

[4 marks] [*2008, No.18*]

Answer :

(a) (b)

60 The diagram shows a circle with centre O.



PR and QR are tangent to the circle at points P and Q respectively. It is given that the length of minor arc PQ is 4 cm and $OR = \frac{5}{\alpha}$ cm. Express in terms of α

(a) the radius, r, of the circle, (*Ans* : $\frac{4}{\alpha}$)

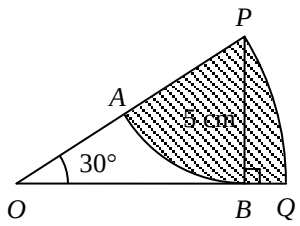
(b) the area, A, of the shaded region. (*Ans* : $\frac{12-8\alpha}{\alpha^2}$)

[4 marks] [*2018, No.12*]

Answer :

(a) (b)

61 The diagram shows a sector of a circle *OPQ* with centre *O* and sector *APB* with centre *P*.

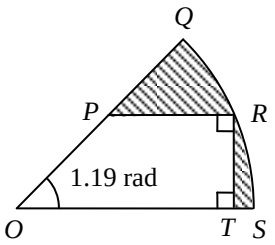


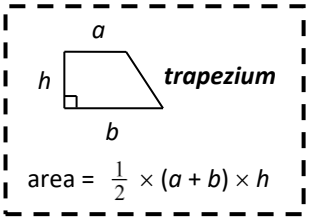
It is given that *A* lies on *OP*, *B* lies on *OQ* and triangle *OPB* is a right angled at *B*. Calculate the area, in cm^2, of the shaded region. [Use $\pi = 3.142$] (*Ans* : 17.62)

[4 marks] [***Forecast***]

Answer :

62 The diagram shows a sector *OQS* of a circle with centre *O* and radius 13 cm. Given *OPRT* is a trapezium with *OT* = 12 cm and *PR* = 2*RT*.



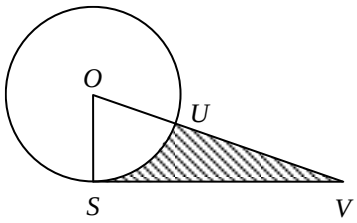


Find the area, in cm^2, of the shaded region. [use $\pi = 3.142$] (*Ans* : 45.56)

[4 marks] [***Forecast***]

Answer :

63 The diagram shows the cross section of a cylindrical roller with centre *O* and radius 20 cm resting on a horizontal ground *SV*. *OUV* is a straight line that represent the handle of the roller and *OU* : *OV* = 1 : 3.

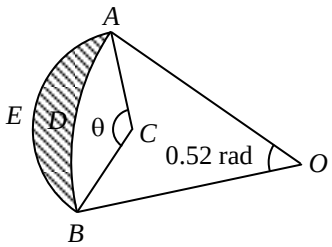


calculate the area, in cm^2, of the shaded region. [use $\pi = 3.142$] (*Ans* : 319.5)

[4 marks] [***Forecast***]

Answer :

64 The diagram shows sector *OADB* of a circle with centre *O* and sector *CAEB* with centre *C*.



It is given that *OA* = 7 cm and *CA* = 2 cm. Using $\pi = 3.142$, calculate

(a) the value of θ, in radians, (*Ans* : 2.238)

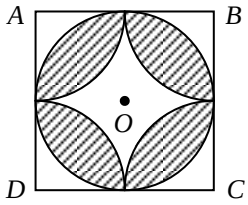
(b) the area, in cm^2, of the shaded region. (*Ans* : 2.338)

[4 marks] [***Forecast***]

Answer :

(a) (b)

65 The diagram shows a square *ABCD* of side 10 cm, a circle with centre *O*, and four quadrants with centre *A*, *B*, *C* and *D*.

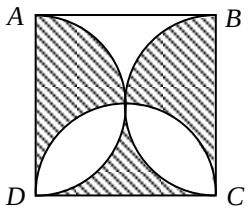


Find the area, in cm^2, of the shaded region. Give your answer in term of π. (*Ans* : $50\pi - 100$)

[3 marks] [***Forecast***]

Answer :

66 The diagram shows a square *ABCD* of side 10 cm, and three semicirlcles.



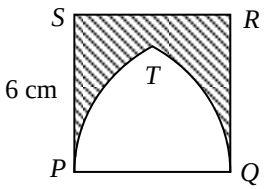
Find the area, in cm^2, of the shaded region. Give your answer in term of π. (*Ans* : $100 - 12.5\pi$)

[3 marks] [***Forecast***]

Answer :

⇒ ***problems solving 3 ~ HOTS***

67 The diagram shows part of the front view of a square shape mural art on a wall in a school building. *PT* is an arc of a circle with a centre *Q* and *QT* is an arc of a circle with a centre *P*.

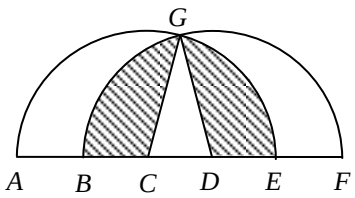


The shaded region shows the part that needs to be repainted. Cheng and his friends decided to paint the area with red colour. Calculate the area, in m, of that region. [use π= 3.142] (*Ans* : 13.89)

[4 marks] [*2014, No.20*]

Answer :

68 The diagram shows two identical semicircles *ABCDEG* and *BCDEFG* with centres *C* and *D* respectively.

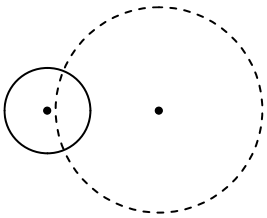


Given that the semicircles have a radius of 8 cm, and *AB* = *BC* = *CD* = *DE* = *EF*. Find the area, in cm^2, of the shaded region. [Use π = 3.142] (*Ans* : 53.37)

[4 marks] [***Forecast***]

Answer :

69 Emily's garden has two fountains which will produce water based on circular movement with radii 5 m and 12 m respectively, as shown in the diagram.



It is given that the distance between both fountain is 13 cm. Find the area of the garden which get the water.

[Use π = 3.142] (*Ans* : 504.74)

[4 marks] [***Forecast***]

Answer :

PAPER 2

⇒ ***Part A → 6 – 8 marks***

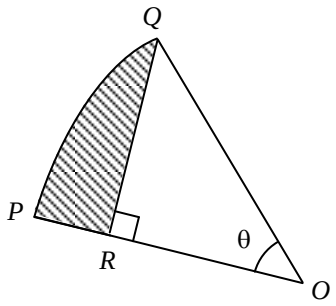
==========

1.4 Circle Application of Circular Measures

1.4.1 Solve problems involving circular measure.

==========

70 The diagram shows the sector *POQ*, with centre *O* and a radius of 10 cm.



The point *R* is on *OP* such that *OR* : *OP* = 3 : 5. Using $\pi = 3.142$, calculate

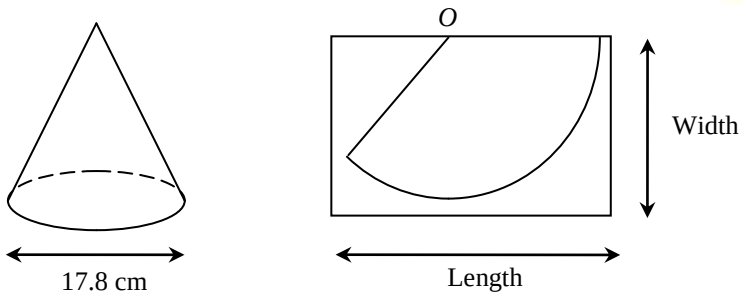
(a) the value of θ, in rad, (*Ans* : 0.9273) [3 marks]

(b) the area of the shaded region, in cm^2. (*Ans* : 22.365) [4 marks]

[***2003, No.4***]

Answer :

71 Susan wantS to make a cap in the shape of a cone as shown in diagram (a). The height of the cone is 24 cm.



(a) (b)

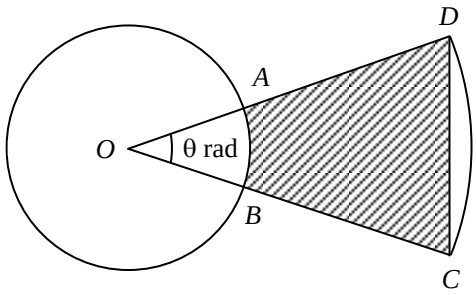
Diagram (b) shows the net of the cone in the form of sector of a circle with centre *O* drawn on a rectangular card. [Use $\pi = 3.142$]

(a) Calculate the minimum value, in cm, of length and of width of the card to the nearest integer.
(*Ans* : length = 41, width = 26) [5 marks]

(b) Hence, find the area, in cm^2, of the card unused. (*Ans* : 350.22) [3 marks]
[***2015, No.6***]

Answer :

72 The diagram shows a circle and a sector of a circle with a common centre *O*. The radius of the circle is *r* cm.



It is given that the length of arc *AB* and arc *CD* are 2cm and 7 cm respectively. *BC* = 10 cm.
[Use $\pi = 3.142$]

Find

(a) the value of *r* and of θ. (*Ans* : $r = 4$, $\theta = \frac{1}{2}$) [3 marks]

(b) the area, in cm^2, of the shaded region. (*Ans* : 42.984) [5 marks]
[***2017, No.3***]

Answer :

73 Mathematics Society of SMK Muhibah organised a competition to design a logo for the society.

The diagram shows the circular logo designed by Amar. The three black coloured regions are congruent, It is given that the perimeter of the black coloured region is 20π. [Use $\pi = 3.142$]

Find

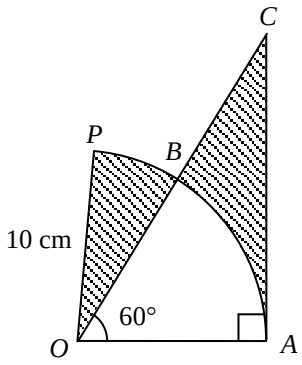
(a) the radius, in cm, of the logo to the nearest integer, (*Ans* : 10) [3 marks]

(b) the area, in cm^2, of the region that are shaded with leaning line. (*Ans* : 75.61) [4 marks]

[***2018, No.5***]

Answer :

74 The diagram shows a sector POA with centre O.



It is given that the length of arc PB is 2.56 cm. [Use $\pi = 3.142$]

Calculate

(a) $\angle POB$ in radians, (*Ans* : 0.256) [2 marks]

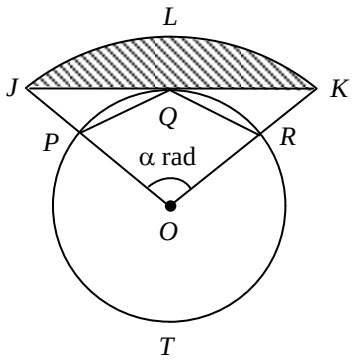
(b) the area, in cm^2, of the shaded region. (*Ans* : 47.055) [4 marks]

[***2019, No.3***]

Answer :

⇒ ***Part B → 10 marks***

75 The diagram shows a circle *PQRT*, centre *O* and radius 5 cm. *JQK* is a tangent to the circle at *Q*. The staright lines, *JO* and *KO*, intersect the circle at *P* and *R* respectively. *OPQR* is a rhombus. *JKL* is an arc of a circle, centre *O*.



Using $\pi = 3.142$, calculate

(a) the angle, α, in term of π, (*Ans* : $\frac{2}{3}\pi$) [2 marks]

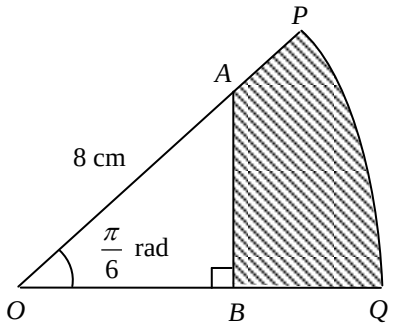
(b) the length, in cm, of the arc *JLK*, (*Ans* : 20.95) [4 marks]

(c) the area, in cm^2, of the shaded region. (*Ans* : 61.44) [4 marks]

[***2004, No.9***]

Answer :

76 The diagram shows a sector POQ of a circle, centre O. The point A lies on OP, the point B lies on OQ and AB is perpendicular to OQ. The length of OA = 8 cm and $\angle POQ = \frac{\pi}{6}$ radian.



It is given that $OA : OP = 4 : 7$. Calculate (Use $\pi = 3.142$)

(a) the length, in cm, of AP, (*Ans* : 6) [1 mark]

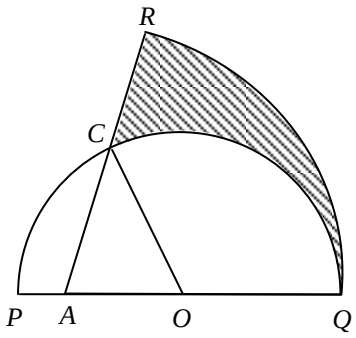
(b) the perimeter, in cm, of the shaded region, (*Ans* : 24.403) [5 marks]

(c) the area, in cm^2, of the shaded region. (*Ans* : 37.46) [4 marks]

[***2005, No.10***]

Answer :

77 The diagram shows the plan of a garden. *PCQ* is a semicircle with centre *O* and has a radius of 8m. *RAQ* is sector of a circle with centre *A* and has a radius of 14m.



Sector *COQ* is a a lawn. The shaded region is a flower bed and has to be fenced. It is given that *AC* = 8 m and $\angle COQ$ = 1.956 radians. [Use π = 3.142]

(a) the area, in m^2, of the lawn, (*Ans* : 62.59) [2 marks]

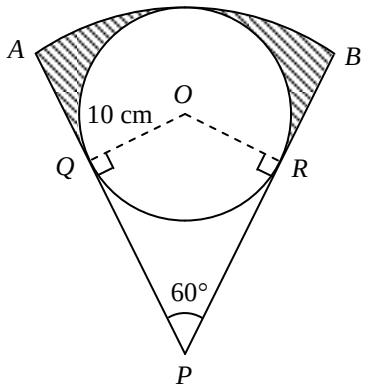
(b) the length, in m, of the fence required for fencing the flower bed. (*Ans* : 38.25) [4 marks]

(c) the area, in m^2, of the flower bed. (*Ans* : 31.39) [4 marks]

[***2006, No.10***]

Answer :

78 The diagram shows a circle, centre O and radius 10 cm inscribed in a sector APB of a circle, centre P. The straight lines, AP and BP, are tangents to the circle at point Q and point R, respectively.



Calculate [Use $\pi = 3.142$]

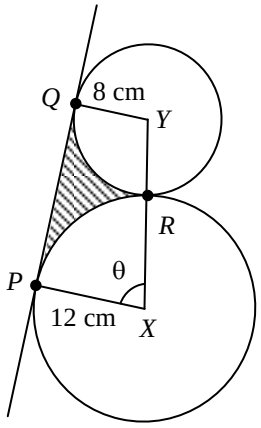
(a) the length, in cm, of the arc AB, (*Ans* : 31.42) [5 marks]

(b) the area, in cm^2, of the shaded region. (*Ans* : 88.63) [5 marks]

[***2007, No.9***]

Answer :

79 The diagram shows two circles. The larger circle has centre *X* and radius 12 cm. The smaller circle has centre *Y* and radius 8 cm. The circle touch at point *R*. The straight line *PQ* is a common tangent to the circles at point *P* and point *Q*.



[Use $\pi = 3.142$]

Given that $\angle PXR = \theta$ radians,

(a) show that $\theta = 1.37$ (to two decimal places) [2 marks]

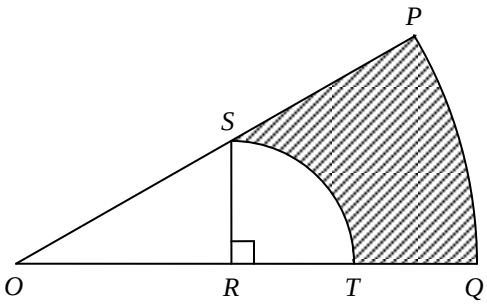
(b) calculate the length, in cm, of the minor arc *QR*, (*Ans* : 14.18) [3 marks]

(c) calculate the area, in cm^2, of the shaded region. (*Ans* : 40.62)[5 marks]

[***2008, No.9***]

Answer :

80 In the diagram, *POQ* is a sector of a circle with centre *O* and radius 26 cm. *SRT* is a quadrant of a circle with centre *R* and radius 5 cm.



It is given that *S* is the midpoint of *OP*. Use $\pi = 3.142$ and give the answer correct to two decimal places. Calculate

(a) $\angle POQ$, in radians, (*Ans* : 0.39) [2 marks]

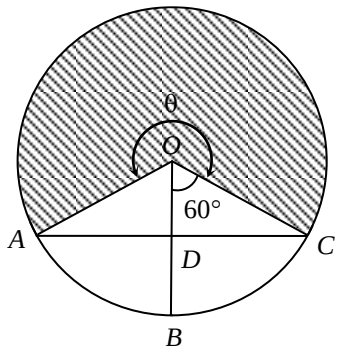
(b) the perimeter, in cm, of the shaded region, (*Ans* : 40.00) [4 marks]

(c) the area, in cm^2, of the shaded region. (*Ans* : 82.18) [4 marks]

[***2009, No.10***]

Answer :

81 The diagram shows a circle with centre O. D is the midpoint of AC and $\angle DOC = 60°$.



It is given that the area of the major sector OAC is $209\frac{7}{15}$ cm^2, Find [Use $\pi = 3.142$]

(a) the value of θ, in radians, (*Ans* : $\frac{4}{3}\pi$ / 4.189) [1 mark]

(b) radius, in cm, of the circle, (*Ans* : 10) [2 marks]

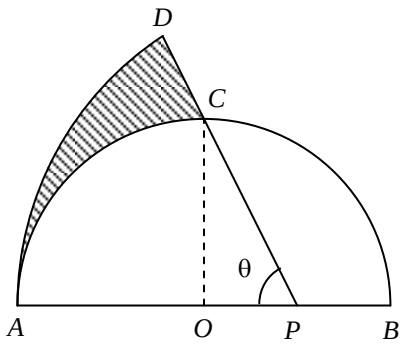
(c) the perimeter, in cm, of the shaded region, (*Ans* : 61.89) [4 marks]

(d) the area, in cm^2, of segment ABC. (*Ans* : 61.43) [3 marks]

[***2010, No.11***]

Answer :

82 In the diagram, *AOBC* is a semicircle with centre *O* and radius 4 cm. *APD* is a sector of circle with centre *P* and radius 6 cm.



It is given that *OC* is perpendicular to *AOB*. Calculate [Use $\pi = 3.142$]

(a) the value of θ, in radians, (*Ans* : 1.107) [2 marks]

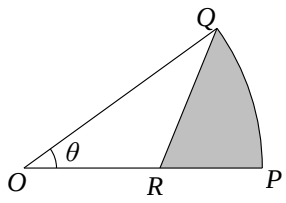
(b) the perimeter, in cm, of the shaded region, (*Ans* : 14.45) [4 marks]

(c) the area, in cm^2, of the shaded region. (*Ans* : 3.358) [4 marks]

[***2011, No.9***]

Answer :

83 The diagram shows a sector POQ of a circle, centre O with radius 16 cm. Point R lies on OP.



It is given that $OR = RQ = 10$ cm. Calculate [Use $\pi = 3.142$]

(a) the value of θ, in radians, correct to three decimal places, (*Ans* : 0.644) [2 marks]

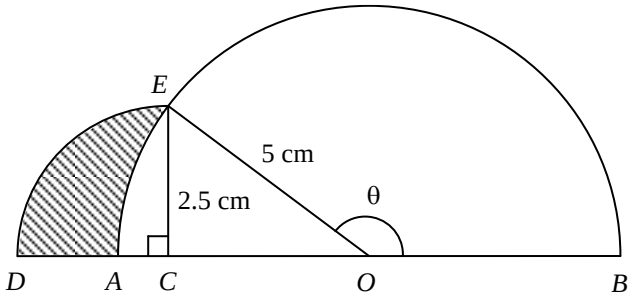
(b) the perimeter, in cm, of the shaded region, (*Ans* : 26.304) [3 marks]

(c) the area, in cm^2, of the shaded region. (*Ans* : 34.40) [5 marks]

[***2012, No.9***]

Answer :

84 The diagram shows a semicircle *AEB*, with centre *O* and quadrant of a circle *DCE*, with centre *C*.



[Use $\pi = 3.142$]

Calculate

(a) the value of θ, in radians, (*Ans* : 2.618) [2 marks]

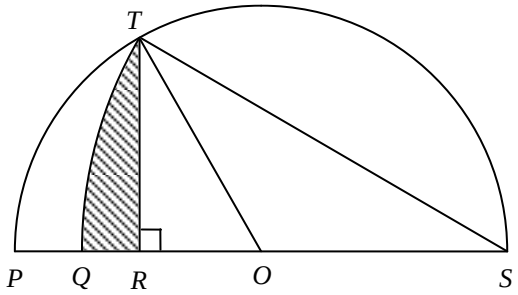
(b) the perimeter, in cm, of the whole diagram, (*Ans* : 28.85) [4 marks]

(c) the area, in cm^2, of the shaded region. (*Ans* : 3.772) [4 marks]

[***2013, No.11***]

Answer :

85 The diagram shows a semicircle PTS with centre O and radius 8 cm. QST is a sector of a circle with centre S and R is the midpoint of OP.



[Use $\pi = 3.142$]

Calculate

(a) $\angle TOR$, in radians, (*Ans* : 1.047) [2 marks]

(b) the length, in cm, of the arc TQ, (*Ans* : 7.256) [4 marks]

(c) the area, in cm^2, of the shaded region. (*Ans* : 8.684) [4 marks]

[***2014, No.11***]

Answer :

FORECAST

⇒ ***Part A → 7 – 8 marks***

86 Alyanna receives a souvenir from her friend in the form of a paper fan with a ribbon around the outside of the paper as shown in the diagram.

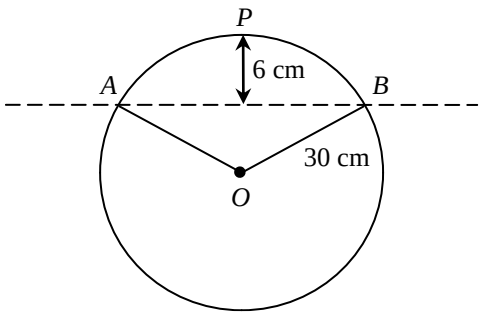
The length of the paper fan is 16 cm. The ratio between the length of wooden part to the length of the paper part is 1: 3. Deva opens the paper fan by 120°. Use $\pi = 3.142$, find

(a) the total length, in cm, of the ribbon for the opened paper fan, (*Ans* : 65.894) [3 marks]

(b) the area, in cm^2, of the paper part. (*Ans* : 251.36) [3 marks]

Answer :

87 The diagram shows the cross section of a log floating in water. The cross section is a circle, with centre O and of radius 30 cm.



The chord AB is at the water surface level and the highest point P is 6 cm above water surface. Use $\pi = 3.142$, find

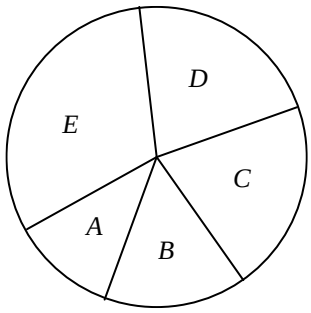
(a) minor $\angle AOB$, in radians, (*Ans* : 1.287) [2 marks]

(a) the length, in cm, of the arc APB, (*Ans* : 38.61) [2 marks]

(b) the cross section area, in cm^2, under the water surface. (*Ans* : 2680.65) [4 marks]

Answer :

88 Floria plans to create a game which requires a circular disc. The dics is divided into five parts as shown in the diagram.



The ratio of the areas of sector $A : B : C : D$ is $1 : 2 : 4 : 5$. Given the length of arc of sector E is 104.5 cm with its subtended angle 2.09 rad. The sector D is to be painted red. A tin of red paint can be used to paint an area of $600\ cm^2$. (Use $\pi = 3.142$)

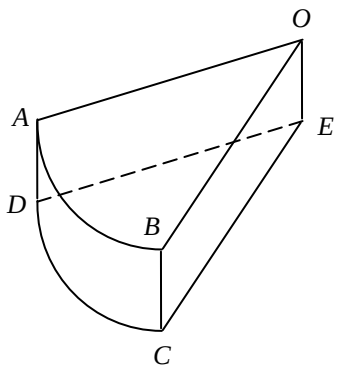
(a) Determine the minimum number of tins of paint to be used by to paint sector D.

(*Ans* : 4) [5 marks]

(b) Find the perimeter of sector B. (*Ans* : 134.95) [2 marks]

Answer :

89 Ann Marie has a cake shop. She wants to do research for the improvement of his cake shop. The diagram shows a piece of cake with uniform cross-section in the shape of a sector *OAB*, of radius 20 cm.



Given the total surface area of the cake is 740 cm^2 and the thickness of the cake is 8 cm. Find

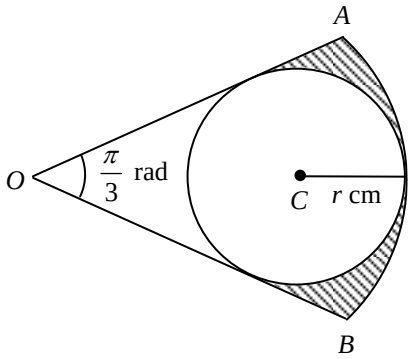
(a) the angle of the sector, in radians, (*Ans* : 0.75) [4 marks]

(b) the perimeter, in cm, of the curved surface of the cake. (*Ans* : 46) [2 marks]

Answer :

⇒ ***Part B→ 10 marks***

90 The diagram shows a circle with centre *C* and of radius *r* cm inscribed in a sector *OAB* of a circle with centre *O* and of radius 42 cm.



Given that $\angle AOB = \frac{\pi}{3}$ rad. Using $\pi = 3.142$, find

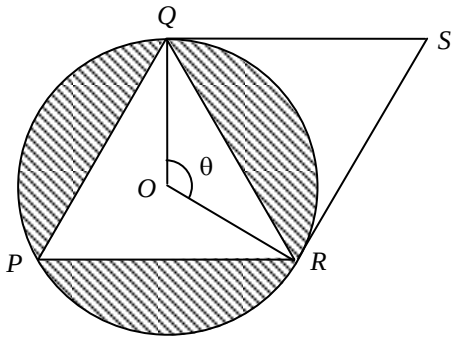
(a) the value of *r*, (*Ans* : 14) [2 marks]

(b) the perimeter, in cm, of the shaded region, (*Ans* : 138.14) [4 marks]

(c) the area, in cm^2, of the shaded region. (*Ans* : 173.71) [4 marks]

Answer :

91 The daigram shows a circle *PQR* with radius 5 cm. *RS* and *QS* are tangent to the circle and $\angle ROQ = \theta$. Given that *PQR* is an equilateral triangle.



Find [Use $\pi = 3.142$]

(a) the value of θ, in degrees, (*Ans* : 120) [1 mark]

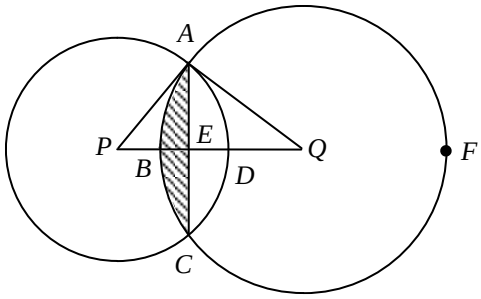
(b) the length, in cm, of *OS*, (*Ans* : 10) [2 marks]

(c) the area, in cm^2, of the whole diagram, (*Ans* : 95.66) [4 marks]

(d) perimeter, in cm, of the shaded region. (*Ans* : 57.40) [3 marks]

Answer :

92 The diagram shows two circles with centres *P* and *Q* respectively. *PBEDQ* is a straight line. *PA* and *QA* are tangent to the circle at point *A*.



Given *PA* = 3 cm and *PQ* = 5 cm, find [Use $\pi = 3.142$]

(a) $\angle PQA$, in radians, (*Ans* : 0.6436) [2 marks]

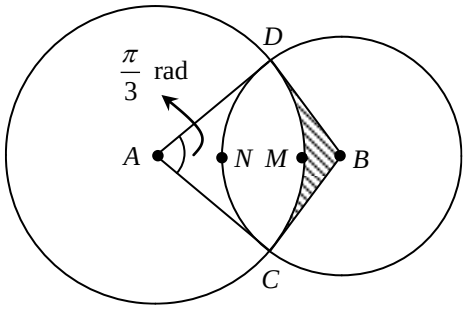
(b) the length, in cm, of the arc *AFC*, (*Ans* : 19.99) [2 marks]

(c) the perimeter, in cm, of *PBA*. (*Ans* : 6.574) [2 marks]

(d) the area, in cm^2, of the shaded region, (*Ans* : 2.616) [4 marks]

Answer :

93 The diagram shows two circles with centres A and B which intersect each other at point C and D. AC and AD are tangents to the circle with centre B.



Given that AB = 6 cm and $\angle CAD = \frac{\pi}{3}$ radian.

[Use π = 3.142. Given your answer correct to three decimal places]

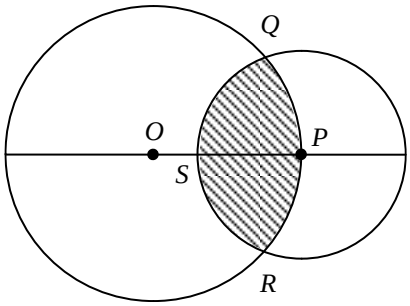
(a) the length, in cm, of AC, (*Ans* : 5.196) [2 marks]

(b) the length, in cm, of arc DNC, (*Ans* : 6.285) [4 marks]

(c) the area, in cm^2, of the shaded region, (*Ans* : 1.450) [4 marks]

Answer :

94 The diagram shows two circles. The radius of the circle centre *O* and circle centre *P* are 8 cm and 4 cm respectively. *OP* is a straight line.



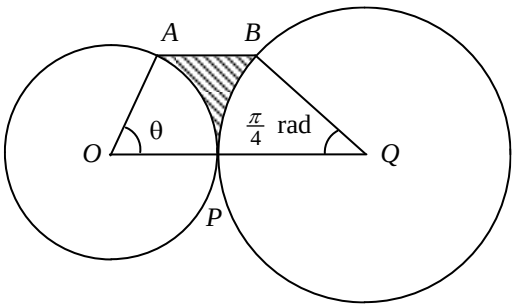
Find [Use $\pi = 3.142$]

(a) the perimeter, in cm, of the shaded region, (*Ans* : 18.63) [5 marks]

(b) the area, in cm^2, of the shaded region, (*Ans* : 22.46) [5 marks]

Answer :

95 The diagram shows two circles with centres *O* and *Q*. The circle with centre *O* has a radius of 4 cm while the circle with centre *Q* has a radius of 5 cm. Two circle touch at *P* and *OPQ* is a straight line. *AB* is parallel to *OPQ*.



Find [Use $\pi = 3.142$]

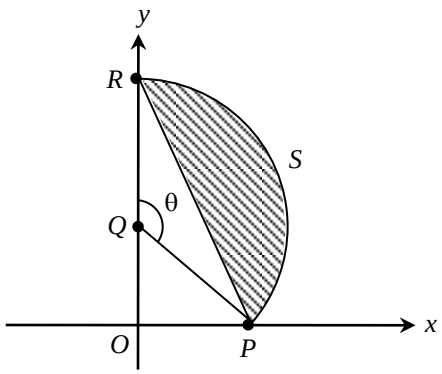
(a) the value of θ, in radians, (*Ans* : 1.084) [3 marks]

(b) the length, in cm, of *AB*, (*Ans* : 3.595) [3 marks]

(c) the perimeter, in cm, of the shaded region, (*Ans* : 11.86) [4 marks]

Answer :

96 In the diagram, *QRSP* is a sector of a circle with centre *Q*. The equation of the straight line *PQ* is $3x + 4y = 24$.



Find [Use $\pi = 3.142$]

(a) the coordinates of points *P* and *Q*, [*Ans* : *P* (8, 0), *Q* (0, 6)] [2 marks]

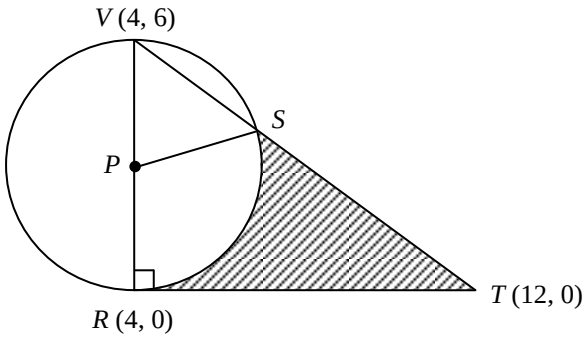
(b) the value of θ, in radians, (*Ans* : 2.215) [2 marks]

(c) the perimeter of the shaded region, (*Ans* : 40.04) [3 marks]

(d) the area, in unit^2, of the shaded region, (*Ans* : 70.77) [3 marks]

Answer :

97 The diagram shows a circle with centre *P*. *RV* and *RT* are diameter and tangent to the circle respectively. Point *S* lies on the circumference of the circle and *VST* is a straight line.



Using $\pi = 3.142$, find

(a) the equation of the locus of a point *M* (*x*, *y*) which moves along the circumference of the circle.

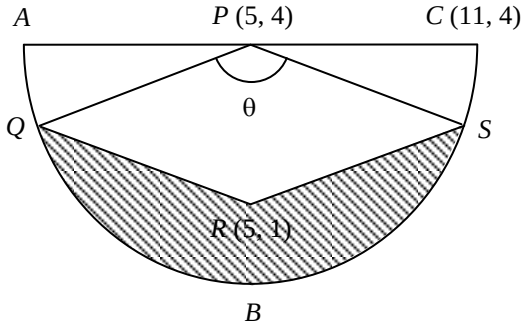
(*Ans* : $x^2 + y^2 - 8x - 6y + 16 = 0$) [3 marks]

(b) the angle *VPS*, in radian, (*Ans* : 1.287) [3 marks]

(c) the area, in unit^2, of the shaded region. (*Ans* : 11.33) [4 marks]

Answer :

98 The diagram shows a semicircle *AQBSC* with centre *P*, and a rhombus *PQRS*.



Given that the coordinates of points *C*, *P* and *R* are (11, 4), (5, 4) and (5, 1) respectively. Find

(a) the radius of the semicircle, (*Ans* : 6) [1 mark]

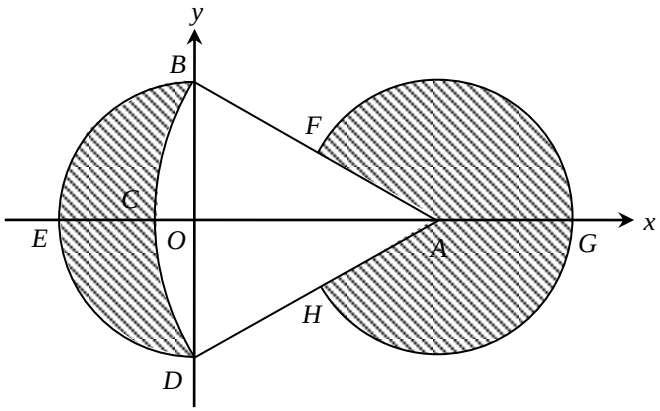
(b) the angle θ, in radians, (*Ans* : 2.636) [3 marks]

(c) the area, in unit^2, of sector *PQBS*, (*Ans* : 47.45) [2 marks]

(d) the area, in unit^2, of the shaded region. (*Ans* : 30.02) [4 marks]

Answer :

99 In the diagram, the coordinates of points A, B, and D are $(3, 0)$, $(0, 2)$ and $(0, -2)$ respectively.



Given $ABCD$ is a sector with centre A, $OBED$ is a semicircle with centre O, and FGH is a major sector of a circle with centre A and of radius 2 cm. Using $\pi = 3.142$, find

(a) $\angle BAD$, in radians, (*Ans* : 1.176) [2 marks]

(b) the area, in unit^2, of the segment $BODC$, (*Ans* : 1.644) [4 marks]

(c) the area, in unit^2, of the shaded region. (*Ans* : 14.85) [4 marks]

Answer :

DIFFERENTIATION

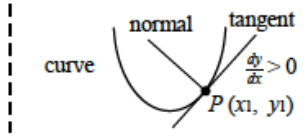
- ONE PAGE NOTE (OPN)
- WORKSHEET

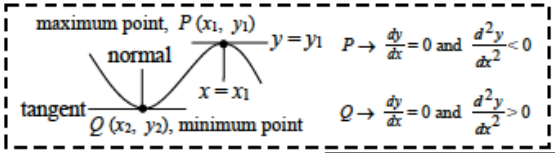
Encik Hafizi Fazli Bakar

ONE PAGE NOTES
" DIFFERENTIATION "

gradient function $= \frac{dy}{dx} = f'(x)$

rate of change of radius $= \frac{dj}{dt}$





LIMIT

- $= f(a) \lim_{x \to a} f(x) = f(a)$
  ~ if $f(a)$ undefined → factorise
- if $0 < a < 1 \rightarrow \lim_{n \to \infty} (a)^n = 0$
- $\lim_{x \to \infty} \frac{1}{x} = 0$ @ $\lim_{x \to \infty} \frac{a}{x} = 0$
- if $\lim_{x \to \infty} f(x) = \frac{\infty}{\infty}$ ~ limit can't be

obtained → divide each term in $f(x)$ by the highest power of x

《 APPLICATION 1 》
EQUATION OF TANGENT/NORMAL

to a point (x_1, y_1) at a curve y

(1) $\frac{dy}{dx}$ → m_1 (gradient / gradient of tangent)

(2) x_1 ↔ y_1

(3) $y = m_1 x + c$ (equation of tangent

(4) $m_1 \times m_2 = -1$ (m_2 = gradient of normal)

(5) $y = m_2 x + c$ (equation of normal

《 APPLICATION 2 》
TURNING POINT/STATIONARY POINT

(minimum point / maximum point)

(1) $\frac{dy}{dx}$ → turning point (x_1, y_1)

(2) $\frac{dy}{dx} = 0$ → x_1 → y_1

(3) $\frac{d^2y}{dx^2}$ → $\frac{d^2y}{dx^2} > 0$ [minimum]

$\frac{d^2y}{dx^2} < 0$ [maximum]

《 APPLICATION 4 》
SMALL CHANGE

(1) $\frac{dy}{dx}$ ↔ information tambahan

(2) x_1 ↔ y_1

(3) decide δx @ δy

(4) $\frac{\delta y}{\delta x} = \frac{dy}{dx}$

(5) $y_{new} = y_1 + \delta y$

(6) $\%\, \delta y = \frac{\delta y}{y_1} \times 100$

FIRST DERIVATIVE
" *FIRST PRINCIPLE* "

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

@

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

NOTES :

- tangent // x-axis → gradient, $\frac{dy}{dx}$, $m_1 = 0$
- tangent // to a straight line → line : m_1
- tangent ⊥ to a straight line → line : m_2
- normal // to a straight line → line : m_2
- normal ⊥ to a straight line → line : m_1

《 APPLICATION 3 》
RATE OF CHANGE

(1) $\frac{dy}{dx}$

(2) x_1 ↔ y_1 additional information

(3) given $\frac{dx}{dt}$ → $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

given $\frac{dy}{dt}$ → $\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$

FIRST DERIVATIVE
" *FORMULA 1* "

$y = f(x)$ $\xrightarrow[\text{derivative}]{\text{first}}$ $\frac{dy}{dx} = f'(x)$ $\xrightarrow[\text{derivative}]{\text{second}}$ $\frac{d^2y}{dx^2} = f''(x)$

$\frac{d}{dx}(k) = 0$

$\frac{d}{dx}(kx) = k$

$\frac{d}{dx}(kx^n) = (nk)x^n$

$\frac{d}{dx}\left(\frac{k}{x^n}\right) = \frac{-nk}{x^{n+1}}$

$\frac{d}{dx}[k(ax+b)^n] = (nk)(ax+b)^{n-1}(a)$

$\frac{d}{dx}\left(\frac{k}{(ax+b)^n}\right) = \frac{-nk(a)}{(ax+b)^{n+1}}$

EXAMPLE 1 :

$y = 3x^4 - \frac{x}{5} + 8$

→ $\frac{dy}{dx} = 12x^3 - \frac{1}{5}$

→ $\frac{d^2y}{dx^2} = 36x^2$

EXAMPLE 2 :

$f(x) = (3x-2)^5 + \frac{3}{x^2}$

→ $f'(x) = 5(3x-2)^4(3) - \frac{6}{x^3}$

→ $f''(x) = 60(3x-2)^3(3) + \frac{18}{x^4}$

FIRST DERIVATIVE
" *FORMULA 2* "

$y = uv$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$y(u)$ & $u(x)$ chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

WORKSHEET
TOPIC 2 : DIFFERENTIATION
[1 – 3 questions → 4 – 7 marks]

==========

2.1 Limit and its Relation to Differentiation

2.1.1 Investigate and determine the value of limit of a function when its variable approaches zero.

==========

⇒ limit 1 ~ direct substitution

1 Find the value of :

(a) $\lim_{x \to 1} (7 - x^2)$, [1 mark] [*2018, No.5a*]

(b) $\lim_{n \to 1} \frac{1+n}{n}$, [1 mark] [*Forecast*]

(c) $\lim_{x \to 0} \frac{3k}{kx+2k}$. [1 mark] [*Forecast*]

Answer :

(a)

(b)

(c)

2 Find the value of :

(a) $\lim_{x \to -2} \sqrt{10-3x}$, [1 mark]

(b) $\lim_{n \to 4} \frac{5-\sqrt{n}}{n+2}$, [1 mark]

(c) $\lim_{x \to 0} \frac{x-4}{\sqrt{x}-2}$. [1 mark]

[*Forecast*]

Answer :

(a)

(b)

(c)

MIND think :

- $\lim_{x \to a} f(x) = f(a)$, where $f(a) \neq \frac{0}{0}$

  ~ if $f(a) = \frac{0}{0}$ → factorisation

  → rationalising the numerator @ denominator of the function

⇒ limit 2 ~ factorization, rationalising the numerator @ denominator of the function

3 Find the value of :

(a) $\lim_{n \to 2} \frac{4 - n^2}{2 - n}$, (*Ans* : 4) [2 marks]

(b) $\lim_{n \to 3} \frac{n^2 - 3n}{2n - 6}$. (*Ans* : $\frac{3}{2}$) [2 marks]

[*Forecast*]

Answer :

(a) (b)

4 Find the value of :

(a) $\lim_{n \to \frac{1}{2}} \frac{2n^2 + 5n - 3}{2n - 1}$, (*Ans* : $3\frac{1}{2}$) [2 marks]

(b) $\lim_{x \to 3} \frac{x^2 - 4x + 3}{2x^2 - 5x - 3}$. (*Ans* : $\frac{2}{7}$) [2 marks]

[*Forecast*]

Answer :

(a) (b)

5 (a) Given that $\lim\limits_{x\to 3} \dfrac{x^2-k}{hx-15} = \dfrac{6}{5}$, find the values of h and k.

(*Ans* : $h = 5$, $k = 9$) [3 marks]

(b) Given $\lim\limits_{x\to -3} \dfrac{x^2+2x+h}{kx+6} = -2$, find the values of $h + k$.

(*Ans* : -1) [3 marks]

[*Forecast*]

Answer :

(a) (b)

6 Find the value of :

(a) $\lim\limits_{n\to 0} \dfrac{\sqrt{n+1}-1}{n}$, (*Ans* : $\frac{1}{2}$) [2 marks]

(b) $\lim\limits_{n\to 1} \dfrac{\sqrt{n+3}-2}{n-1}$. (*Ans* : $\frac{1}{4}$) [2 marks]

[*Forecast*]

Answer :

(a) (b)

7 Find the value of :

(a) $\lim\limits_{x\to 0} \dfrac{2x}{3-\sqrt{x+9}}$, (*Ans* : −12) [2 marks]

(b) $\lim\limits_{x\to 3} \dfrac{6-5x+x^2}{2-\sqrt{x+1}}$. (*Ans* : −4) [2 marks]

[*Forecast*]

Answer :

(a) (b)

⇒ ***limit 3***

8 Find the value of :

(a) $\lim\limits_{n\to\infty} (0.5)^n$, (b) $\lim\limits_{n\to\infty} \left(\dfrac{2}{5}\right)^n$.

Answer :

(a) (b)

MIND think :

- if $-1 < a < 1 \rightarrow \lim\limits_{n\to\infty} (a)^n = 0$
- $\lim\limits_{x\to\infty} f(x) = \frac{\infty}{\infty}$ ~ limit can't be obtained → divide each term in $f(x)$ by the highest power of x

  NOTE : $\lim\limits_{x\to\infty} \frac{1}{x} = 0$ @ $\lim\limits_{x\to\infty} \frac{a}{x} = 0$

$\Rightarrow$ *limit 4 ~ limit can't be obtained* $\rightarrow$ *divide each term in f (x) by the highest power of x*

9 Find the value of :

(a) $\lim\limits_{n\to\infty} \dfrac{3}{6+n}$, [1 mark]

(b) $\lim\limits_{n\to\infty} \dfrac{3n}{5-n}$. (*Ans* : -3) [2 marks]

[*Forecast*]

Answer :

(a) (b)

10 Find the value of :

(a) $\lim\limits_{n\to\infty} \dfrac{1+n^2}{3+2n^2}$. (*Ans* : $\frac{1}{2}$) [2 marks]

(b) $\lim\limits_{n\to\infty} \dfrac{2n^3+n+1}{4n^2+5n^3+1}$. (*Ans* : $\frac{2}{5}$) [2 marks]

[*Forecast*]

Answer :

(a) (b)

$\Rightarrow$ *existence of imit*

11 The function f is defined by, $f(x) = \begin{cases} \dfrac{x-2}{x+3}, & 0 \le x < 1 \\ 1+ax^2, & x \ge 1 \end{cases}$

(a) Given that $\lim\limits_{x\to 1} f(x)$ exist, find the value of a. [*Ans* : $-\frac{5}{4}$) [2 marks]

(b) With this value of a, determine whether f is continuous at $x = 1$. [1 mark]

Answer :

[*Forecast*]

(a) (b)

12 The function f is defined by, $f(x) = \begin{cases} x + a\,, & x \le -2 \\ bx^2 + 1\,, & -2 < x \le 1 \\ 3x + 1\,, & x > 1 \end{cases}$

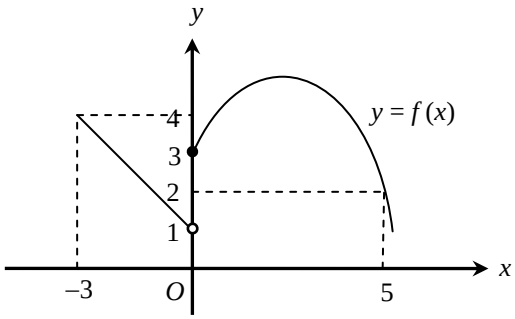
If $\lim\limits_{x \to -2} f(x)$ and $\lim\limits_{x \to 1} f(x)$ exist, find the values of a and b.

(*Ans* : $a = 15$, $b = 3$)

[3 marks] [*Forecast*]

Answer :

13 The diagram shows a part of the function graph $y = f(x)$.



Based on the graph :

(a) Find

(i) $\lim\limits_{x \to -3} f(x)$, [1 mark]

(ii) $\lim\limits_{x \to 5} f(x)$. [1 mark]

(b) (i) Find $f(0)$. [1 mark]

(ii) Determine whether $\lim\limits_{x \to 0} f(x)$ exist. Give reason for your answer.

[2 marks] **[***Forecast***]**

Answer :

(a) (i)

(ii)

(b) (i)

(ii)

MIND think :

$\lim_{x \to a} f(x)$ exist if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ $\rightarrow$ f is continuous at $x = a$

==========

2.1.2 Determine the first derivative of a function $f(x)$ by using the first principle.

==========

14 Given $y = \frac{1}{4}x^2$. Find $\frac{dy}{dx}$ using the first principle. (*Ans* : $\frac{1}{2}x$)

[3 marks] [*Forecast*]

Answer :

15 Find the first derivative of $y = \frac{4}{x}$ from first principle. (*Ans* : $-\frac{4}{x^2}$)

[3 marks] [*Forecast*]

Answer :

16 Find the first derivative of $y = \frac{3}{x^2} + 5x$ from first principles. (*Ans* : $\frac{dy}{dx} = -\frac{6}{x^3} + 5$)

[3 marks] [*Forecast*]

Answer :

17 A particle move along a straight line such that the displacement of the particles from a fixed point *O* is *s* m. The particle moving from point *A* with $s(t) = 3t^2 - 2t + 5$ where *t* is the time, in seconds. By using first principles, find the velocity of the particle when *t* = 3. (*Ans* : $\frac{dy}{dx} = 6x - 2$, 16)

[4 marks] [*Forecast*]

Answer :

MIND think :

- $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ @ $f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$
- Gradient of the curve is also known as "***gradient of the tangent***"

==========

2.2 The First Derivative

2.2.1 Derive the formula of first derivative inductively for the functiony = ax^n, *a* is a constant and *n* is an integer.

==========

18 Complete each of the following :

$\frac{d}{dx}(a) = 0$	$\frac{d}{dx}(ax) = k$	$\frac{d}{dx}(ax^n) = (an)x^{n-1}$	$\frac{d}{dx}\left(\frac{a}{x^n}\right) = \frac{-an}{x^{n+1}}$
$y = 1 \rightarrow \frac{dy}{dx} =$	$y = x \rightarrow \frac{dy}{dx} =$	$y = x^2 \rightarrow \frac{dy}{dx} =$	$\frac{d}{dx}\left(\frac{1}{x}\right) =$
$y = 2 \rightarrow \frac{dy}{dx} =$	$y = 2x \rightarrow \frac{dy}{dx} =$	$y = 2x^3 \rightarrow \frac{dy}{dx} =$	$\frac{d}{dx}\left(\frac{3}{x^2}\right) =$
$y = -3 \rightarrow \frac{dy}{dx} =$	$y = -3x \rightarrow \frac{dy}{dx} =$	$y = -\frac{3x^4}{7} \rightarrow \frac{dy}{dx} =$	$\frac{d}{dx}\left(-\frac{4}{x^3}\right) =$
$y = -\frac{1}{5} \rightarrow \frac{dy}{dx} =$	$y = \frac{x}{8} \rightarrow \frac{dy}{dx} =$	$y = \frac{1}{5}\sqrt{x} \rightarrow \frac{dy}{dx} =$	$\frac{d}{dx}\left(\frac{2}{\sqrt[3]{x^2}}\right) =$

==========

2.2.2 Determine the first derivative of an algebraic function.

==========

19 (a) Given that $y = \frac{2}{3}x^6 - \frac{x^4}{2} + \frac{1}{5}$, find $\frac{dy}{dx}$. (*Ans* : $4x^5 - 2x^3$)

(b) Given that $f(x) = \frac{1}{2}x^4 - 7x$, find $f'(x)$. (*Ans* : $2x^3 - 7$)

Answer :

(a) (b)

20 (a) Given that $y = 2x + \frac{2}{x}$, find $\frac{dy}{dx}$. (*Ans* : $2 - \frac{2}{x^2}$)

(b) Given that $f(x) = \frac{1}{2x^3} - \frac{1}{x^2} + 3$, find $f'(x)$. (*Ans* : $-\frac{3}{2x^4} + \frac{2}{x^3}$)

Answer :

(a) (b)

21 (a) Given that $f(x) = \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt[3]{x}} - 5$, find $f'(x)$. (*Ans* : $\frac{3}{4\sqrt{x}} - \frac{2}{3x^{\frac{4}{3}}}$)

(b) Given that $y = \frac{x^3 - 5}{x^2}$, find $\frac{dy}{dx}$. (*Ans* : $1 + \frac{10}{x^3}$)

Answer :

(a) (b)

22 (a) Differentiate $(3x^2 - 1)(7x^3 - 4)$ with respect to x. (*Ans* : $105x^4 - 24x - 21x^2$)

(b) Differentiate $x(\sqrt{x} + 9x + 2)$ with respect to x. (*Ans* : $\frac{3}{2}x^{\frac{1}{2}} + 18x + 2$)

Answer :

(a) (b)

23 (a) Find $\frac{d}{dx}\left(2+\frac{1}{x^2}\right)^2$. (*Ans* : $-\frac{8}{x^3}-\frac{4}{x^5}$)

(b) Find $\frac{d}{dx}\left(\frac{(2+\sqrt{x})(2+x^2)}{\sqrt{x}}\right)$. (*Ans* : $-\frac{2}{x^{\frac{3}{2}}}+3\sqrt{x}+2x$)

Answer :

(a) (b)

24 (a) Given that $r^3h = 216$, find $\frac{dh}{dr}$. (*Ans* : $-\frac{648}{r^4}$)

(b) Given that $3x^2 - x + 4xy = 0$, find $\frac{dy}{dx}$. (*Ans* : $-\frac{3}{4}$)

Answer :

(a) (b)

25 Given that the function $f(x) = hx^3 + \frac{k}{x^2}$ has a gradient function $f'(x) = 2x^2 - \frac{64}{x^3}$, where a and b are constants. Find the value of h and k. (*Ans* : $h = \frac{2}{3}$, k =32)

[3 marks] [*Forecast*]

Answer :

2.2.3 Determine the first derivative of composite function.

26 Complete each of the following :

$\frac{d}{dx}[\,k\,(ax+b)^n] = (nk)\,(ax+b)^{n-1}(a)$	$\frac{d}{dx}\left[\frac{k}{(ax+b)^n}\right] = \frac{(-nk)\,(a)}{(ax+b)^{n+1}}$
$\frac{d}{dx}[\,(x-5)^2\,] =$	$\frac{d}{dx}\left[\frac{1}{x-5}\right] =$
$\frac{d}{dx}[\,2(3x+5)^3\,] =$	$\frac{d}{dx}\left[\frac{2}{(3x+5)^2}\right] =$
$\frac{d}{dx}[\,-3(2-x)^4\,] =$	$\frac{d}{dx}\left[-\frac{3}{(2-x)^3}\right] =$
$\frac{d}{dx}\left[-\frac{2}{7}(2-3x)^5\right] =$	$\frac{d}{dx}\left[\frac{1}{\sqrt{x^3}-5x}\right] =$

27 Differentiate each of the following with respect to x using "*chain rule*" :
[*compare with the answer in question 26*]

(a) $y = -3(2-x)^4$

(b) $y = \frac{2}{(3x+5)^2}$

Answer :

(a) (b)

28 Given that $f(x) = \dfrac{3}{2(x^2+4)^3}$, evaluate $f'(-1)$. (*Ans* : $\frac{9}{625}$)

[3 marks] [*Forecast*]

Answer :

29 Given that $y = \sqrt{5-2x}$. Find the value of $\dfrac{dy}{dx}$ when $y = 2$. (*Ans* : $-\frac{1}{2}$)

[3 marks] [*Forecast*]

Answer :

30 It is given that $y = \dfrac{2}{3}u^7$, where $u = 3x - 5$. Find $\dfrac{dy}{dx}$ in terms of x. [*Ans* : $14(3x-5)^6$]

[3 marks] [*2006, No.18*]

Answer :

31 Given $x = t^2 + 3$ and $\dfrac{dy}{dt} = 14t^3$, find

(a) $\dfrac{dx}{dt}$,

(b) $\dfrac{dy}{dx}$, in terms of x. [*Ans* : $7(x-3)$]

[4 marks] [*2014, No.18*]

Answer :

(a) (b)

MIND think :

$y(u)$ ~ y in terms of u **and** $u(x)$ ~ u in terms of x $\xrightarrow{\textbf{chain rule}}$ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

==========

2.2.4 Determine the first derivative of a function involving product and quotient of algebraic expressions.

==========

⇒ ***product rule***

32 Differentiate $3x^2(2x-5)^4$ with respect to x. [*Ans* : $\frac{dy}{dx} = 6x(6x-5)(2x-5)^3$]

[3 marks] [*2004, No.20*]

Answer :

33 Find the gradient of the curve $y = x\sqrt{x^2+3}$ when $x = 1$. (*Ans* : $\frac{5}{2}$)

[3 marks] [*Forecast*]

Answer :

34 Given that $f(x) = (2x - 3)^2 (5x + 1)^3$, evaluate $f'\left(\frac{1}{2}\right)$. (*Ans* : 392)

[3 marks] [*Forecast*]

Answer :

MIND think :

product rule **~ y = uv** $\rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ @ $\frac{d}{dx}(uv) = uv' + vu'$, where *u* and *v* are in terms of *x*

$\Rightarrow$ ***quotient rule***

35 Given that $h(x) = \frac{7x^2 + 8}{1 - 6x}$, find the value of $h'(1)$. (*Ans* : 0.8)

[3 marks] [*Forecast*]

Answer :

36 Given that $f(x) = \frac{\sqrt{7 - 3x}}{2 - x}$, find the value of $f'(1)$. (*Ans* : $\frac{5}{4}$)

[3 marks] [*Forecast*]

Answer :

37 Given that $\frac{d}{dx}\left(\frac{2x-5}{x+3}\right) = \frac{k}{(x+3)^2}$, where k is constant. Find the value of k. (*Ans* : 11)

[3 marks] [*Forecast*]

Answer :

MIND think :

quotient rule ~ $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ @ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$, where u and v are in terms of x

38 Given that $\frac{x^2}{y} = 48 + x$, find $\frac{dy}{dx}$. [*Ans* : $\frac{x(96+x)}{(48+x)^2}$]

[3 marks] [*Forecast*]

Answer :

39 Given that $\frac{1}{u} + \frac{1}{v} = \frac{1}{12}$, find $\frac{dv}{du}$. [*Ans* : $-\frac{144}{(u-12)^2}$]

[3 marks] [*Forecast*]

Answer :

40 Given that $y = \frac{x-2}{5+x^2}$, find $\frac{dy}{dx}$. Hence, find the range of the value of x such that all the values of y and $\frac{dy}{dx}$ are negative. (*Ans* : $x < -1$)

[4 marks] [*Forecast*]

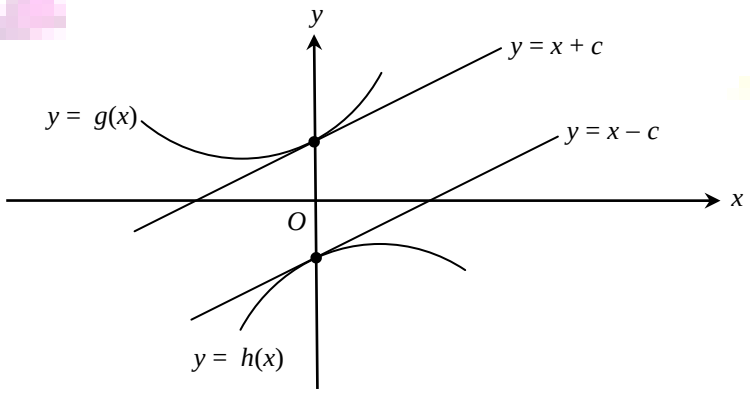
Answer :

41 Given that $f(x) = \frac{4x-3}{x^2+1}$, find the range of the value of x such that $f(x)$ and $f'(x)$ are both positive. (*Ans* : $\frac{3}{4} < x < 2$)

[4 marks] [*Forecast*]

Answer :

42 The diagram shows two curve $y = h(x)$, $y = g(x)$ and the tangent ot each curve when $x = 0$.



It is given that $f(x) = \dfrac{h(x)}{g(x)}$, find $f'(0)$ in term of c. (*Ans* : $\frac{2}{c}$)

[3 marks] [*Forecast*]

Answer :

==========

2.3 The Second Derivative

2.3.1 Determine the second derivative of an algebraic function.

==========

43 Given that $h(x) = \dfrac{1}{(3x-5)^2}$, evaluate $h''(1)$. (*Ans* : $\frac{27}{8}$)

[4 marks] [*2005, No.19*]

Answer :

44 Given the function $h(x) = kx^3 - 4x^2 + 5x$, find

(a) $h'(x)$,

(b) the value of k if $h''(1) = 4$. (*Ans* : 2)

[4 marks] [*2012, No.19*]

Answer :

(a) (b)

45 Find the value of $f''(2)$ if $f'(x) = 2x^3 - 4x + 3$. (*Ans* : 20)

[3 marks] [*2018, No.5b*]

Answer :

46 Given that $y = 3x - \frac{2}{x}$. Show that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

[3 marks] [*Forecast*]

Answer :

47 Given that $y = x^2 - 4x + 1$. Solve the equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - y = 2x + 1$. (*Ans* : 2, $\frac{8}{3}$)

[4 marks] [*Forecast*]

Answer :

48 Given that $y = \frac{2x+1}{x-4}$. Solve the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$. (*Ans* : 6)

[4 marks] [*Forecast*]

Answer :

MIND think :

$$y = f(x) \xrightarrow[\text{derivative}]{\text{first}} \frac{dy}{dx} = f'(x) \xrightarrow[\text{derivative}]{\text{second}} \frac{d^2y}{dx^2} = f''(x)$$

==========

2.4 Application of Differentiation

2.4.1 Interpret gradient of tangent to a curve at different points.

2.4.2 Determine equation of tangent and normal to a curve at a point.

2.4.3 Solve problems involving tangent and normal.

==========

⇒ gradient of tangent / equations of tangent at a point on a curve

49 The curve $y = f(x)$ is such that $\frac{dy}{dx} = 3kx + 5$, where k is a constant. The gradient of the curve at $x = 2$ is 9. Find the value of k. (*Ans* : $\frac{2}{3}$)

[2 marks] [*2007, No.19*]

Answer :

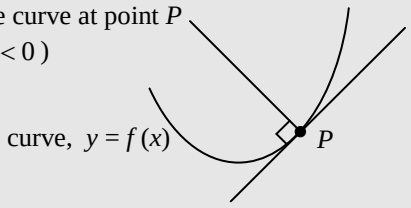
50 The gradient of the tangent to the curve $y = x^2(2 + px)$ at $x = -2$ is 7. Find the value of p. (*Ans* : $\frac{5}{4}$)

[3 marks] [*2012, No.20*]

Answer :

MIND think :

normal to the curve at point P
($m_2 < 0$)



tangent to the curve at point P
($\frac{dy}{dx} > 0$ @ $m_1 > 0$)

- equation of tangent / normal, at a point (x_1, y_1) on a curve, $y = f(x)$

(1) $\frac{dy}{dx}$ ⟷ m_1 (gradient / gradient of tangent)

(2) x_1 ⟷ y_1

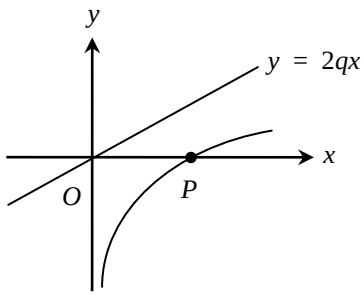
(3) $y = m_1x + c$ (equation of tangent)

(4) $m_1 \times m_2 = -1$ (m_2 = gradient of normal)

(5) $y = m_2x + c$ (equation of normal)

NOTE :
- tangent // x–axis → gradient, $\frac{dy}{dx}$, $m_1 = 0$
- tangent // a line → line : m_1
- tangent ⊥ a line → line : m_2
- normal // a line → line : m_2
- normal ⊥ a line → line : m_1

51 The diagram shows a part of curve $y = \frac{2x-6}{x+2}$ and a straight line.



It is given that the straight line is parallel to the tangent of the curve at point P. Find the value of q. (*Ans* : $\frac{1}{5}$)

[4 marks] [*2016, No.7*]

Answer :

52 The gradient of the curve $y = \frac{a}{x} + bx^2$ at the point $(3, 6)$ is 7. Find the values of a and b. (*Ans* : $a = -9$, $b = 1$)

[3 marks] [*Forecast*]

Answer :

53 Find the coordinates of the points on the curve $y = \frac{1}{3}x^3 - 3x^2 + 7x + 1$ such that the tangen to the curve at that point is perpendicular to the line $2y = x + 4$. [*Ans* : (3, 4)]

[3 marks] [*Forecast*]

Answer :

54 The equation of a curve is $y = 3x^2 - px + 2$, where p is a constant. The tangent to the curve at the point where $x = 2$ passes through (5, 5). Find the value of p. (*Ans* : 9)

[3 marks] [*Forecast*]

Answer :

55 Find the equation tangent to the curve $y = 3x^2 - 2x + 1$ which is parallel to x-axis. (*Ans* : $y = \frac{2}{3}$)

[3 marks] [*Forecast*]

Answer :

56 Given the equation of a curve is $xy = 12$. Find the equation of the tangent to the curve when apabila $x = 2$. (*Ans* : $y = -3x + 12$)

[3 marks] [*Forecast*]

Answer :

57 The tangent to the curve $y = \sqrt{1+2x}$ at the point $P(4, 3)$ intersect the x-axis at point Q. Find the distance of PQ. (*Ans* : $3\sqrt{10}$)

[4 marks] [*Forecast*]

Answer :

58 The tangent to the curve $y = ax^3 + bx + 2$ at $\left(1, \frac{1}{2}\right)$ is parallel to the normal to the curve $y = 4 + 6x + x^2$ at $(-2, -4)$. Find the values of a and b. (*Ans* : $a = \frac{1}{2}$, $b = -2$)

[4 marks] [*Forecast*]

Answer :

⇒ *gradient of normal / equations of normal at a point on a curve*

59 The point P lies on the curve $y = (x - 5)^2$. It is given that the gradient of the normal at P is $-\frac{1}{4}$. Find the coordinate of P. [*Ans* : (7, 4)]

[3 marks] [*2006, No.17*]

Answer :

60 The normal to the curve $y = x^2 - 5x$ at point P is parallel to the straight line $y = -x + 12$. Find the equation of the normal to the curve at point P. (*Ans* : $y = -x - 3$)

[4 marks] [*2008, No.20*]

Answer :

61 The point $P(1, -5)$ lies on the curve $y = 3x^2 - 8x$. Find

(a) the gradient of the tangent to the curve at point P, (*Ans* : -2)

(b) the equation of the normal to the curve at point P. (*Ans* : $\frac{1}{2}x - \frac{11}{2}$)

[4 marks] [*2013, No.19*]

Answer :

(a) (b)

62 If the equation of the normal to the curve $y = hx + \frac{k}{x}$ at the point (2, 7) is $2x + y - 11 = 0$, find the values of h and k. (*Ans* : $h = 2$, $k = 6$)

[4 marks] [*Forecast*]

Answer :

63 The straight line $x + 4y - 10 = 0$ is a normal to the curve $y = (2x - 3)^2 - 4$ at point Q. Find the equation of the tangent to the curve at point Q. (*Ans* : $y = 4x - 11$)

[4 marks] [*Forecast*]

Answer :

64 The tangent to the curve $y = x^2 - 3x + 4$ at (1, 3) intersects the normal to the curve at (3, 4), at the point M. Find the coordinates of point M. [*Ans* : (–, 6)]

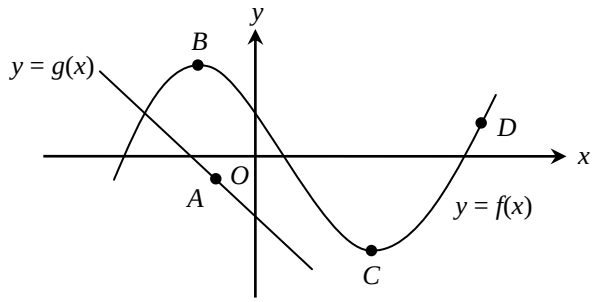
[4 marks]

[*Forecast*]

Answer :

2.4.4 Determine the turning points and their nature.

65 The diagram shows a cubic graph $y = f(x)$ and a linear graph $y = g(x)$.



Point A lies on the straight line. Points B, C and D lies on the curve. The tangents to the curve at point B and point C are parallel to the x-axis. State which point(s) that satisfies the following condition :

(a) $\frac{dy}{dx} = 0$.

(b) $\frac{dy}{dx} < 0$.

(c) $\frac{d^2y}{dx^2} > 0$.

[2 marks] [*2017, No.1*]

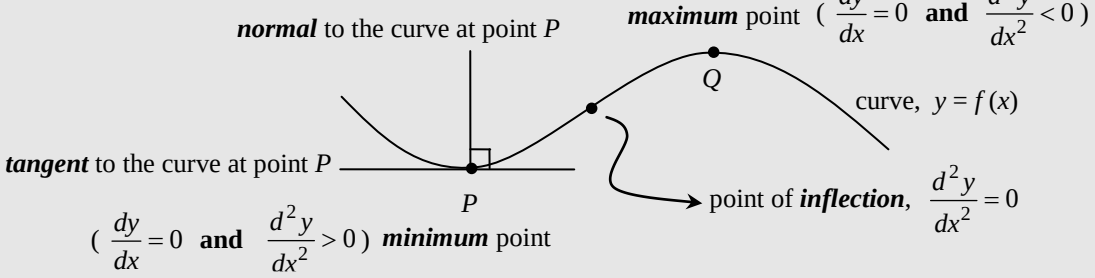
Answer :

(a)

(b)

(c)

MIND think :



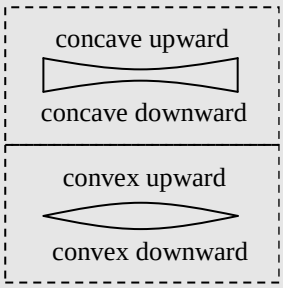
- turning point @ stationary point → maximum point @ minimum point

(1) $\frac{dy}{dx}$ turning point $(x_1,\ y_1)$

(2) $\frac{dy}{dx} = 0 \longrightarrow x_1 \longrightarrow y_1$

(3) $\frac{d^2y}{dx^2}$ ← $\frac{d^2y}{dx^2} > 0$ [minimum] $\frac{d^2y}{dx^2} = 0$ [inflection point]

$\frac{d^2y}{dx^2} < 0$ [maximum]



66 Given that $y = 14x(5 - x)$, calculate

(a) the value of x when y is maximum, (*Ans* : $\frac{5}{2}$)

(b) the maximum value of y. (*Ans* : $\frac{175}{2}$)

[3 marks] [***2003, No.15***]

Answer :

(a) (b)

67 The curve $y = x^2 - 32x + 64$ has a minimum point at $x = p$, where p is a constant. Find the value of p. (*Ans* : 16)

[3 marks] [*2007, No.20*]

Answer :

68 Given $y = 2x(x - 6)$, find

(a) $\frac{dy}{dx}$, (*Ans* : $4x - 12$)

(b) the value of x when y is minimum, (*Ans* : 3)

(c) the minimum value of y. (*Ans* : -18)

[3 marks] [*2010, No.20*]

Answer :

(a) (b) (c)

69 The curve $y = px^4 + 2x$ has turning point at $(-1, q)$. Find the value of p and of q. (*Ans* : $p = \frac{1}{2}$, $q = -\frac{3}{2}$) [4 marks]

[3 marks] [*2019, No.17*]

Answer :

70 The curve $y = mx + \frac{n}{2x-1}$ has a turning point at (2, 7). Find the values of m and n.

(*Ans* : $m = 2$, $n = 9$)

[4 marks] [*Forecast*]

Answer :

71 The curves $y = 2x^2 - 4x + 5$ and $y = x^3 - px^2 + x + q$ have a common turning point. Find the values of p and q. (*Ans* : $p = 2$, $q = 3$)

[4 marks] [*Forecast*]

Answer :

72 Given $p + q = 12$ and $y = 2p^2 - q^2$. Find the minimum value of y. (*Ans* : -288)
[4 marks] [*Forecast*]

Answer :

==========

2.4.5 Solve problems involving maximum and minimum values and interpret the solutions.

==========

73 Due to the high living cost, Siva has planted several types of vegetables for his own consumption on a rectangular shape empty plot of land behind his house. He plans to fence the land which has a dimension of $6x$ m and $(4 - x)$ m.

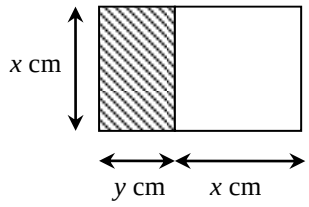
Find the length, in m, the fence he has to buy when the area of the land is maximum. (*Ans* : 28)
[4 marks] [*2014, No.17*]

Answer :

74 Zainal has a rectangular piece of zinc with a perimeter of 25 cm. He wants to use that piece of zinc to build an open cylinder at both ends. Find the length and the width, in cm, of the piece of zinc that makes the volume of the cylinder is maximum. (*Ans* : $\frac{25}{3}$, $\frac{25}{6}$)
[4 marks] [*2015, No.23*]

Answer :

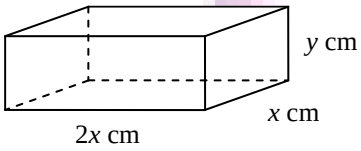
75 A piece of wire of length 10 cm is cut into six pieces and placed to enclose the diagram as shows in below.



4 pieces of the wire are x cm in length and 2 pieces of the wire are y cm in length. Find the perimeter of the shaded region if the area of the shaded region is maximum. (*Ans* : 7.5)
[4 marks] [*Forecast*]

Answer

76 The diagram shows a cuboid of volume 72 cm^3.



(a) Show that the total surface area of the cuboid, A cm^2, is given by $A = 4x^2 + \frac{216}{x}$.

(b) Hence, find the minimum total surface area, in cm^2, of the cuboid. (*Ans* : 108)

[4 marks] [*Forecast*]

Answer :

(a) (b)

2.4.6 Interpret and determine rates of change for related quantities.

==========

77 Two variable, x and y, are related by the equation $y = 3x + \frac{2}{x}$. Given that y increases at a constant rate of 4 units per second, find the rate of change of x when $x = 2$. (*Ans* : 1.6)

[3 marks] [*2004, No.21*]

Answer :

78 Given that $y = \frac{2x-6}{x}$. If y increases at a constant rate of 0.4 unit s^{-1}, find the rate of change of x when $y = 1$. (*Ans* : 2.4)

[3 marks] [*Forecast*]

Answer :

79 Given that $y = x^2 - 5x + 4$. If x decreases from 4 to 3.4 in 3 second, find the rate of change of y. (*Ans* : -0.6)

[3 marks] [*Forecast*]

Answer :

MIND think :

- rates of change

(1) $\frac{dy}{dx}$

(2) $x_1 \leftrightarrow y_1$ — additional information

(3) given $\frac{dx}{dt} \rightarrow \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

given $\frac{dy}{dt} \rightarrow \frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$

2.4.7 Solve problems involving rates of change for related quantities and interpret the solutions.

==========

80 The volume of water, V cm^3, in a container is given by $V = \frac{1}{3}h^3 + 8h$, where h cm is the height of the water in the container. Water is poured into the container at the 10 cm^3 s^{-1}, find the rate of change of the height of water, at the instant when its height is 2 cm. (*Ans* : $\frac{5}{6}$)

[3 marks] [*2005, No.20*]

Answer :

81 A block of ice in the form of a cube with sides x cm, melts at a rate of 9.72 cm^3 per minute. Find the rate of change of x at the instant when $x = 12$ cm. (*Ans* : -0.0225)

[4 marks] [*2009, No.20*]

Answer :

82 The volume of a sphere is increasing at a rate of 12.8π cm^3 s^{-1}. Find the radius of the sphere at the instant when the radius is increasing at a rate of rate of 0.2 cms^{-1}. [volume of sphere, $V = \frac{4}{3}\pi r^3$] (*Ans* : 4)

[3 marks] [*2010, No.21*]

Answer :

83 The surface area of a cube increase at a constant rate of $15\ cm^2s^{-1}$. Find the rate of change of side length, in cms^{-1}, when the volume of the cube is $125\ cm^3$. (*Ans* : 0.25 / $\frac{1}{4}$)

[3 marks] [*2016, No.6*]

Answer :

84 A drop of ink falls on a piece of paper and expands in the shape of a circle. If the radius of the ink increases at a constant rate of 18 mm for every 6 second, find the rate of change in the area of ink at the instant when its radius is 5 mm. (*Ans* : 30π)

[3 marks] [*Forecast*]

Answer :

85 The radius of a circle is increasing at the rate of $0.5\ cm\,s^{-1}$. Find the rate of increase of the area of the circle when its circumference is 25 cm. (*Ans* : 12.5)

[3 marks] [*Forecast*]

Answer :

86 Water flow into a conical vessel of semi vertical angle 45° at the rate of $9\ cm^3\,s^{-1}$. Find the rate at which the water surface is rising at the instant when the depth is 3 cm. (*Ans* : $\frac{1}{\pi}$)

[4marks] [*Forecast*]

Answer :

2.4.8 Interpret and determine small changes and approximations of certain quantities.

2.4.9 Solve problems involving small changes and approximations of certain quantities.

⇒ *small changes in quantities*

87 Given that $y = x^2 + 5x$, use differentiation to find the small change in y when x increases from 3 to 3.01. (*Ans* : 0.11)

[3 marks] [*2003, No.16*]

Answer :

88 It is given that $y = 3x^2 + x - 4$,

(a) find the value of $\frac{dy}{dx}$ when $x = 1$, (*Ans* : 7)

(b) express the approximate change in y, in terms of p, when x changes from 1 to $1 + p$, where p is a small value. (*Ans* : $7p$)

[4 marks] [***2006, No.19***]

Answer :

(a) (b)

MIND think :

- small changes and approximations

(1) $\frac{dy}{dx}$

(2) $x_1 \longleftrightarrow y_1$ additional information

(3) determine δx @ δy

(4) $\frac{\delta y}{\delta x} = \frac{dy}{dx}$

(5) $y_{new} = y_1 + \delta y$

(6) $\%\,\delta x = \frac{\delta x}{x_1} \times 10$

(7) $\%\,\delta y = \frac{\delta y}{y_1} \times 10$

89 Two variables, x and y, are related by the equation $y = \frac{16}{x^2}$. Express, in terms of h, the approximate change in y, when x changes from 4 to $4 + h$, where h is a small value. (*Ans* : $-\frac{h}{2}$)

[3 marks] [*2008, No.19*]

Answer :

90 It is given that $y = 10 - \frac{12}{x}$. Find the small change in x, in terms of p, when the value of y changes form 4 to $4 + p$. (*Ans* : $\frac{p}{3}$)

[3 marks] [*2011, No.20*]

Answer :

91 It is given that $L = 4t - t^2$ dan $x = 3 + 6t$.

(a) Express $\frac{dL}{dx}$ in terms of t. (*Ans* : $\frac{2-t}{3}$)

(b) Find the small change in x, when L changes from 3 to 3.4 at the instant $t = 1$. (*Ans* : 1.2)

[4 marks] [*2018, No.6*]

Answer :

(a) (b)

⇒ *percentage change*

92 Given that $y = 5x^2 - 4x + 3$. If x increases 3 % when $x = 2$, find the percentage of corresponding increase in y. (*Ans* : 6.4)

[4 marks] [*Forecast*]

Answer :

93 Given that $y = 5 - \frac{3}{x}$. If y is increases from 4 with a small value, $\frac{p}{25}$, find in term of p, the percentage of corresponding increase in x. (*Ans* : 4p)

[4 marks] [*Forecast*]

Answer :

⇒ *approximate value using differentiation*

94 Given that $y = \sqrt[3]{x}$, use differentiation to estimate the value of $\sqrt[3]{1002}$. (*Ans* : $10\frac{1}{150}$)

[4 marks] [*Forecast*]

Answer :

95 Use differentiation, determine an approximate value for the volume of a sphere when its radius is 3.02 cm. (*Ans* : 36.72π)

[4 marks] [*Forecast*]

Answer :

96 Given that $y = \frac{1}{2x^2}$. Find

(a) the value of $\frac{dy}{dx}$ when $x = 10$, (*Ans* : -0.001)

(b) the approximate value of $\frac{1}{2(10.5)^2}$. (*Ans* : 0.0045)

[4 marks] [*Forecast*]

Answer :

(a) (b)

⇒ *problems solving*

97 Water is poured into a container. After t second, the height of the water level is x cm. Given that the height of the water level is increasing at the rate of $1.44t$ cm s^{-1}. Find the small change in x when t increases from 4.0 s to 4.1s. (*Ans* : 0.576)

[3 marks] [*Forecast*]

Answer :

98 The height of a cylinder is fixed at 16 cm. Find the approximate increase in the total surface area of the cylinder when its radius increase from 4 cm kepada 4.02 cm. (*Ans* : 0.96π)

[3 marks] [*Forecast*]

Answer :

99 A metal in the shape of a cuboid has a square base of x cm. The height of the cuboid is three times the length of the side of its base. When it is heated, the volume of cuboid increases from 81 cm^3 to 83 cm^3. Find the small change in the length of the side of its base at the instant. (*Ans* : $\frac{2}{81}$ / 0.02469)

[3 marks] [*Forecast*]

Answer :

100 The height of a cone is four times its base radius. Find the approximate change in its base radius if its volume increase from 36π cm^3 to 39π cm^3. (*Ans* : $\frac{1}{12}$)

[3 marks] [*Forecast*]

Answer :

PAPER 2

⇒ *Part A → 6 – 8 marks*

~ *gradient of tangent, equation of tangent / gradient of normal, equation of normal*

101 It is given the equation of the curve is $y = 2x(1 - x)^4$ and the curve passes through $P(2, 4)$. Find

(a) the gradient of the curve at point P, (*Ans* : 18) [4 marks]

(b) the equation of the normal to the curve at point P.

(*Ans* : $y = -\frac{1}{18}x + \frac{37}{9}$) [3 marks]

[***2015, No.2***]

Answer :

~ *turning point → minimum point / value ; maximum point / value*

102 Given the equation of a curve is :

$y = x^2(3 - x) + \frac{1}{2}$

(a) Find the gradient function of the curve. (*Ans* : $6x - 3x^2$) [2 marks]

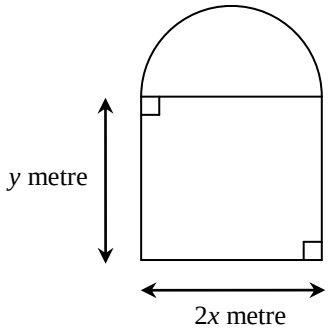
(b) Find the coordinates of the turning points. [*Ans* : $(0, \frac{1}{2})$; $(2, \frac{9}{2})$] [3 marks]

(c) Hence, determine whether each of the turning points is a maximum or minimum.
[*Ans* : $(0, \frac{1}{2})$ minimunm ; $(2, \frac{9}{2})$ maximum] [3 marks]

[***2013, No.5***]

Answer :

103 The diagram shows a front view of a window.



The arc of the window is a semicircle. The perimeter of the window is 4 metre.

(a) Express the front surface area of the window in terms of x and π.

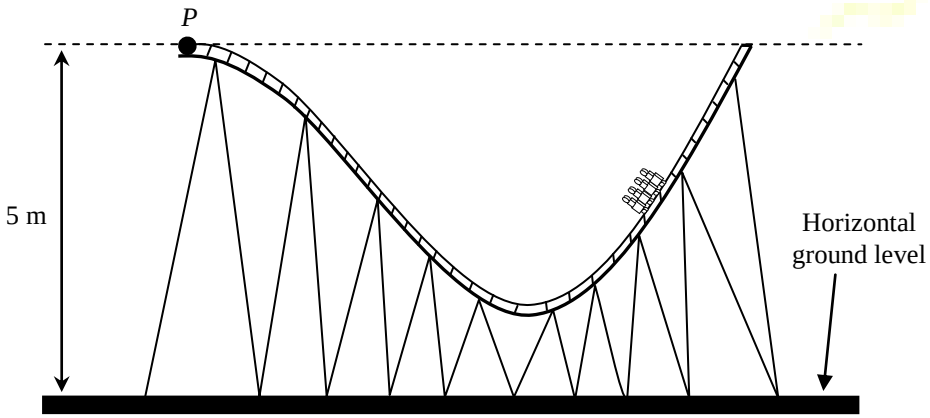
(*Ans* : $A = 4x - 2x^2 - \frac{1}{2}\pi x^2$) [3 marks]

(b) By using $\pi = 3.142$, find the width, in metre, of the window when the front surface area is maximum. (*Ans* : 1.1202) [3 marks]

[***2016, No.6***]

Answer :

104 The diagram shows the front view of a part of a roller coaster track in a miniature park.



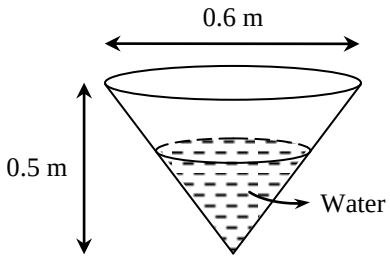
The curve part of the track of the roller coaster is represented by an equation $y = \frac{1}{64}x^3 - \frac{3}{16}x^2$, with point P as the origin. Find the shortest vertical distance, in m, from the track to ground level. (*Ans* : 1)

[6 marks]
[***2018, No.6***]

Answer :

~ *rates of change*

105 The diagram shows a conical container of diameter 0.6 m and height 0.5 m. Water is poured into the container at a constant rate of 0.2 m^3s^{-1}.



Calculate the rate of change of the height of the water level at the instant when the height of the water level is 0.4 m.

(Use $\pi = 3.142$; Volume of a cone = $\frac{1}{3}\pi r^2 h$) (*Ans* : 1.105)

[4 marks] [***2003, No.9a***]

Answer :

~ ***small changes, approximate value***

106 It is given that the equation of a curve is $y = \frac{5}{x^2}$.

(a) Find the value of $\frac{dy}{dx}$ when $x = 3$. (*Ans* : $-\frac{10}{27}$) [3 marks]

(b) Hence, estimate the value of $\frac{5}{(2.98)^2}$. (*Ans* : $\frac{76}{135}$ / 0.563) [4 marks]

[***2017, No.2***]

Answer :

⇒ ***Part B → 10 marks***

~ ***gradient of tangent, equation of tangent / gradient of normal, equation of normal ; turning point ; rates of change ; small changes, approximate value***

107 A metal solid is formed by combining a cone and a cylinder with common radius, r cm. The total surface area of the solid, A cm^2, is given by $A = 2\pi\left(\frac{18}{r} + \frac{r^2}{3}\right)$.

(a) (i) The solid expands when heated. It is given that the surface area of solid changes at the rate of 1.4π cm^2 s^{-1}. Find the rate of change of its radius, in cm s^{-1}, when it radius is 6 cm. (*Ans* : 0.2)

(ii) Find the approximate change in the surface area of the solid, in term of π, when its radius increases from 6 cm to 6.02 cm. (*Ans* : 0.14π)
[6 marks]

(b) Ifa solid of a same shape to be formed in such that the total surface area is minimum, find the minimum total surface area of the solid, in term of π. (*Ans* : 18π)
[4 marks]

[***2019, No.7***]

Answer :

FORECAST

⇒ Part A → 6 – 8 marks

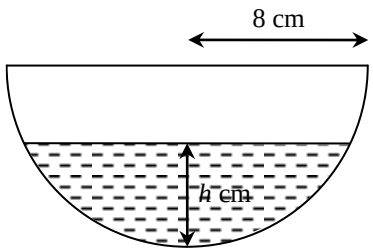
~ maximum / minimum value ; rate of change ; small changes, approximate value

108 (a) The Science and Mathematics Association would like to carry out an herbal cultivation project. The association was given 37 m of wire to fence a rectangular area. If one side of the area to be fenced is a wall. Calculate the appropriate measurement, in m, so that the area of the herb cultivation area is maximum. (*Ans* : 9.25, 18.5) [4 marks]

(b) The radius of a spherical balloon changes from 6 cm to 5.8 cm. Find the approximate change in the volume of the balloon. (*Ans* : -28.8π) [3 marks]

Answer :

109 The diagram shows the cross section of a hemispherical bowl of radius 8 cm.



Water is poured into the bowl with the condition, the high of the water, h, increases at a rate of $0.2\ \text{ms}^{-1}$.

(a) Show that the surface area of the water, $L\ \text{cm}^2$, in the bowl is given by:

$$L = \pi(16h - h^2)$$ [2 marks]

(b) Find the rate of increase of the surface area of the water when h is 6 cm.
(*Ans* : 0.8π) [3 marks]

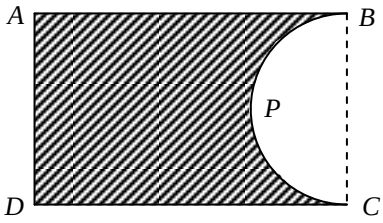
(c) Find the small change in the surface area of the water when the height changes from 2 cm to 2.01 cm. (*Ans* : 0.12π) [2 marks]

Answer :

⇒ ***Part B → 10 marks***

~ ***maximum / minimum value ; rate of change ; small changes, approximate value***

110 (a) The diagram shows a rectangular and semi-circular area fenced with a 120 m long wire.



It is given that $AD = 2x$ metres.

(i) Show that the area of the shaded area, in m^2, is given by $L = 120x - 2x^2 - \frac{3}{2}\pi x^2$.

[2 marks]

(ii) Find the value of x, when L is the maximum.

(*Ans* : 8.938) [3 marks]

(b) A piece of wire 80 cm long is bent to form a circle. When the wire is heated, its length increases with a rate of 0.2 cms^{-1}. [Use $\pi = 3.142$]

(i) Calculate the rate of change, in cms^{-1}, the radius of the circle.

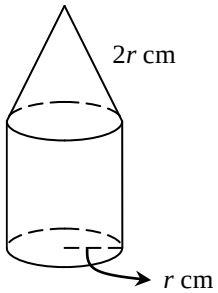
(*Ans* : 0.03183) [3 marks]

(ii) Find the radius, in cm, of the circle after 4 seconds.

(*Ans* : 12.86) [2 marks]

Answer :

111 The diagram shows a solid block consisting of a right cone which is located above a cylinder of radius r cm. The length of slant of the cone is $2r$ cm and the volume of the cylinder is 81π.



(a) Show that the surface area of the block, A cm^2, is given by $A = 3\pi\left(r^2 + \frac{54}{r}\right)$. [2 marks]

(b) Calculate the minimum value for the surface area, in terms of π, of the block.

(*Ans* : 81π) [3 marks]

(c) It is given that A is increasing at a rate of 63π cm^{2}s^{-1}, find the increasing rate of radius whent the radius is 6 cm. (*Ans* : 2) [3 marks]

(d) Find the small change in A when r increases from 6 cm to 6.002 cm.

(*Ans* : 0.063π) [2 markah]

Answer :

INTEGRATION

- ONE PAGE NOTE (OPN)
- WORKSHEET

Encik Lee Chiong Tee

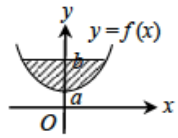
ONE PAGE NOTES
" INTEGRATION "

HINT :

$\int \frac{x^3-2x}{x}\,dx = \int x^2-2\,dx$

$\int \frac{x^2-9}{x+3}\,dx = \int x-3\,dx$

also applied to "DIFFERENTIATION"



volume of shaded area revolved 360° at x-axis = volume of shaded area revolved 180° at x-axis

$= \pi \int_a^b x^2\,dy$

"INTEGRATION" REVERSE "DIFFERENTIATION"

$\frac{d}{dx}(A) = B \;\Leftrightarrow\; \int B\,dx = A$

$y = A$ and $\frac{dy}{dx} = B \;\Leftrightarrow\; \int B\,dx = A$

INTEGRATION
" FORMULAE "

$\int k\,dx = kx + c$

$\int kx^n\,dx = \frac{kx^{n+1}}{n+1} + c,\ n \neq -1$

$\int \frac{k}{x^n}\,dx = \frac{k}{x^{n-1}[-(n-1)]} + c,\ n \neq 1$

$\int k(ax+b)^n\,dx = \frac{k(ax+b)^{n+1}}{(n+1)(a)} + c,\ n \neq -1$

$\int \frac{k}{(ax+b)^n}\,dx = \frac{k}{(ax+b)^{n-1}[-(n-1)(a)} + c,\ n \neq 1$

DEFINITE INTEGRALS

$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$

$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

$\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

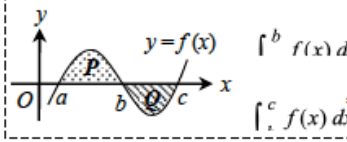
@

$\int_a^b f(x)\,dx = k$

$\Leftrightarrow$

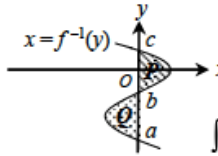
$\int_b^a f(x)\,dx = -k$

AREA



$\int_a^b f(x)\,dx = h \Rightarrow$ area $P = h$

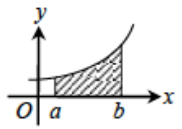
$\int_a^c f(x)\,dx = k \Rightarrow$ area $Q = |k|$



$\int_b^c f^{-1}(y)\,dy = h \Rightarrow$ area $P = h$

$\int_a^b f^{-1}(y)\,dy = k \Rightarrow$ area $Q = |k|$

AREA and VOLUME

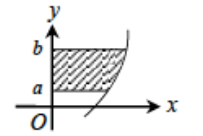


shaded area on x-axis

$\int_a^b y\,dx$

volume of shaded area revolved 360° at x-axis

$\pi \int_a^b y^2\,dx$

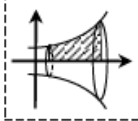


shaded area on y-axis

$\int_a^b x\,dy$

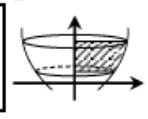
volume of shaded area revolved 360° at y-axis

$\pi \int_a^b x^2\,dy$



revolved

[×] 90° 180° [×]



EXAMPLE 1 :

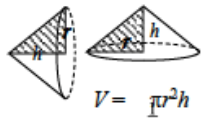
$\int \frac{5x^3}{2} - \frac{4}{\sqrt{x}} + 8\,dx$

$= \frac{5x^4}{2(4)} - \frac{4}{x^{\frac{-1}{2}}(\frac{1}{2})} + 8x + c$

EXAMPLE 2 :

$\int (1-x)^4 + \frac{2}{(3x-5)^3}\,dx$

$= \frac{(1-x)^5}{5(-1)} - \frac{2}{(3x-5)^2(-2)(3)} + c$



$V = \frac{1}{3}\pi r^2 h$

$V = \pi r^2 h$

WORKSHEET
TOPIC 3 : INTEGRATION
[1 – 2 questions → 4 – 8 marks]

3.1 Integration as the Inverse of Differentiation

3.1.1 Explain the relation between differentiation and integration.

1 Solve each of the following :

(a) Given $\frac{d}{dx}\left(\frac{x^2-1}{x-3}\right) = f(x)$, find $\int f(x)\ dx$. [1 mark]

(b) If $y = \frac{x^2}{x-1}$ and $\frac{dy}{dx} = h(x)$, find $\int h(x)\ dx$. [1 mark]

(a) Given $\frac{d}{dx}[f(x)] = 2g(x)$, find $\int g(x)\ dx$. [*Ans* : $\frac{1}{2}f(x)$] [2 marks] [*2013, No.21*]

Answer :

(a) (b) (c)

2 Solve each of the following :

(a) Given that $y = \frac{4x-1}{x+3}$ and $\frac{dy}{dx} = \frac{5}{2}k(x)$, find $\int 3k(x)\ dx$. [*Ans* : $\frac{6}{5}\left(\frac{4x-1}{x+3}\right)$] [2 marks]

(b) Given thtat $\frac{d}{dx}\left(\frac{x^2}{2x-1}\right) = \frac{2x(x-1)}{(2x-1)^2}$, find $\int \frac{x(x-1)}{4(2x-1)^2}\ dx$. [*Ans* : $\frac{x^2}{8(2x-1)}$] [2 marks]

[*Forecast*]

Answer :

(a) (b)

***MIND think* :**

- integration = reverse of differentiation

  → $\frac{d}{dx}(A) = B \leftrightarrow \int B\,dx = A$ @ $\frac{d}{dx}[f(x)] = f'(x) \leftrightarrow \int [f'(x)]\,dx = f(x)$

  → $y = A$ and $\frac{dy}{dx} = B \leftrightarrow \int B\,dx = A = y$

3 (a) Show that $\frac{d}{dx}\left(\frac{x-1}{x^2+3}\right) = \frac{3+2x-x^2}{(x^2+3)^2}$. [2 marks]

(b) Hence, or otherwise, find $\int \frac{3+2x-x^2}{2(x^2+3)^2}\,dx$. [*Ans* : $\frac{x-1}{2(x^2+3)}$] [2 marks]

[*Forecast*]

Answer :

(a) (b)

4 Given that $f(x) = 4(2x+1)^3$ and $f'(x) = m(2x+1)^n$.

(a) Find the value of $m+n$. (*Ans* : 26) [2 marks]

(b) Hence, find the value of $\frac{1}{12}\int f'(x)\,dx$, where $x = 2$. (*Ans* : $\frac{125}{3}$) [2 marks]

[*clon textbook*]

Answer :

(a) (b)

5 Given that $y = 2x^2(1-x)^2$.

(a) Find $\frac{dy}{dx}$. (*Ans* : $4x - 12x^2 + 8x^3$) [2 marks]

(b) If $\int x - 3x^2 + 2x^3\,dx = ky$, find the value of k. (*Ans* : $\frac{1}{4}$) [2 marks]

[*clon textbook*]

Answer :

(a) (b)

6 The profit function from the sale of books of a company M is given by $P = 150t^2 + 40t^3$, where P is the profit obtained, in RM, and t is the time, in day.

(a) Find the rate of profit obtained by the book company after 5 days. (*Ans* : 4500) [2 marks]

(b) Given that the rate of profit obtained from another book company N is given by $\frac{dP}{dt} = 50t + 20t^2$. By using reverse of differentiation, find the number of time the profit obtained by company N compare to the profit obtained by company M. (*Ans* : $\frac{1}{6}$) [2 marks]

[*clon textbook*]

Answer :

(a)

(b)

MIND think :

product rule ~ **y = uv** $\rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ @ $\frac{d}{dx}(uv) = uv' + vu'$, where u and v are in terms of x

quotient rule ~ $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ @ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$, where u and v are in terms of x

3.2 *Indefinite Integral*

3.2.1 Derive the indefinite integral formula inductively.

7 Complete each of the following :

$\int a\ dx = kx + c$	$\int ax^n\ dx = \frac{ax^{n+1}}{n+1} + c$ where $n \neq -1$	$\int \frac{k}{x^n}\ dx = \frac{k}{x^{n-1}[-(n-1)]} + c$ where $n \neq 1$
$\int 1\ dx$ =	$\int x\ dx$ =	$\int \frac{1}{x^2}\ dx$ =
$\int 2\ dx$ =	$\int 5x^2\ dx$ =	$\int \frac{5}{x^3}\ dx$ =
$\int -7\ dx$ =	$\int \frac{5x^3}{9}\ dx$ =	$\int \frac{7}{2x^4}\ dx$ =
$\int \frac{1}{5}\ dx$ =	$\int \frac{13}{7}\sqrt{x}\ dx$ =	$\int -\frac{3}{\sqrt[3]{x}}\ dx$ =

==========

3.2.2 Determine indefinite integral for algebraic functions.

==========

8 Given that $\int (6x^2 + 1)\,dx = px^3 + x + c$, where p and c are constants, find

(a) the value of p, (*Ans* : 2)

(b) the value of c for $\int (6x^2 + 1)\,dx = 13$, when $x = 1$. (*Ans* : 10)

[4 marks] [***2008, No.21***]

Answer :

(a) (b)

9 Find the integral for each of the following :

(a) $\int x\left(2x - \frac{5}{x}\right)dx$ (*Ans* : $\frac{2}{3}x^3 - 5x + c$)

(b) $\int (2x + \sqrt{x})^2\ dx$ (*Ans* : $\frac{4}{3}x^3 + \frac{8}{5}x^{\frac{5}{2}} + \frac{1}{2}x^2 + c$)

Answer :

(a) (b)

10 Find the integral for each of the following :

(a) $\int \frac{(x-3)(x+3)}{x^2}\ dx$ (*Ans* : $x + \frac{9}{x} + c$)

(b) $\int \frac{5x^2 - 3}{\sqrt[3]{x^2}}\ dx$ (*Ans* : $\frac{15}{7}x^{\frac{7}{3}} - 9x^{\frac{1}{3}} + c$

)

Answer :

(a) (b)

11 Find the integral for each of the following :

(a) $\int \frac{4x^2-9}{4x-6}\,dx$ (*Ans* : $\frac{1}{2}x^2+\frac{3}{2}x+c$)

(b) $\int \frac{x^2+3x+2}{x+1}\,dx$. (*Ans* : $\frac{1}{2}x^2+2x+c$

[2 marks]

Answer :

(a)

(b)

==========

3.2.3 Determine indefinite integral for functions in the form of $(ax + b)^n$, where a and b are constants, n is an integer and $n \neq -1$.

==========

12 Complete each of the following :

$\int k(ax+b)^n\,dx = \frac{k(ax+b)^{n+1}}{(n+1)(a)} + c,$ where $n \neq -1$	$\int \frac{k}{(ax+b)^n}\,dx = \frac{k}{(ax+b)^{n-1}[-(n-1)](a)} + c$ where $n \neq 1$
$\int (x-5)^2\,dx$ =	$\int \frac{1}{(x-5)^2}\,dx$ =
$\int 2(3x-5)^3\,dx$ =	$\int \frac{2}{(3x-5)^3}\,dx$ =
$\int 3(5-x)^4\,dx$ =	$\int \frac{-3}{(5-x)^4}\,dx$ =
$\int -4\sqrt{2-x}\,dx$ =	$\int \frac{3}{\sqrt[4]{5-2x}}\,dx$ =

13 Find the indefinite integral for each of the following using "*substituition method*" :

[*compare with the answer in question 12*]

(a) $\int 2(3x-5)^3 \, dx$

(b) $\int \frac{3}{\sqrt[4]{5-2x}} \, dx$

Answer :

(a) (b)

14 Given that $\int \frac{5}{(1-x)^4} dx = k(1-x)^n + c$, find the values of k and n. (*Ans* : $k = \frac{5}{3}$, $n = -3$)

[3 marks] [*2003, No.17*]

Answer :

15 It is given that $\int \frac{5}{(2x+3)^n} dx = \frac{p}{(2x+3)^5} + c$, where c, n and p are constants. Find the value of n and of p. (*Ans* : $n = 6$, $p = -\frac{1}{2}$)

[3 marks] [*2017, No.17*]

Answer :

16 Find the integral for each of the following :

(a) $\int 2x - \frac{1}{x^3} + \frac{1}{(1+2x)^2}\ dx$ (*Ans* : $x^2 + \frac{1}{2x^2} - \frac{1}{2(1+2x)} + c$)

(b) $\int \frac{dx}{2(3-x)^2}$. (*Ans* : $\frac{1}{2(3-x)} + c$)

Answer :

(a)

(b)

==========

3.2.4 Determine the equation of curve from its gradient function.

==========

⇒ ***gradient function → equation of curve***

17 Given $\frac{dv}{dt} = 8t - 3$ and $v = 20$ when $t = 2$, express v in terms of t. (*Ans* : $v = 4t^2 - 3t + 10$)

[3 marks] [*2013, No.20*]

Answer :

18 The gradient function of a curve is $(x - 3)$. It is given $P(2, 1)$ lies on the curve. Find

(a) the gradient of the tangent at point P, (*Ans* : -1)

(b) the equation of the curve. (*Ans* : $y = \frac{1}{2}x^2 - 3x + 5$)

[4 marks] [*2015, No.16*]

Answer :

(a) (b)

19 Given that $3x + 2\frac{dy}{dx} = 4$ and $y = 1$ when $x = 2$, express y in terms of x. (*Ans* : $y = 2x - \frac{3}{4}x^2$)

[3 marks] [*Forecast*]

Answer :

20 Given that $\frac{dv}{dt} = \frac{a}{t^2} + 2$, where a is a constant. If $\frac{dv}{dt} = \frac{2}{3}$ and $v = 5$ when $t = 3$, find the value of v when $t = -2$. (*Ans* : -15)

[4 marks] [*Forecast*]

Answer :

21 The gradient function of a curve passing through the points $(1, 4)$ and $(-2, k)$ is $3x^2 + 4x - 3$. Find the value of k. (*Ans* : 10)

[4 marks] [*Forecast*]

Answer :

22 The slope function of a curve at any point $P(x, y)$ is $kx + 3$, where k is constant. Given that the curve passes through the points $(0, 8)$ and $(1, 15)$, find the value of k. (*Ans* : $k = 8$)
[4 marks] [*Forecast*]

Answer :

23 It is given that the slope function of a curve is $px + q$, where p and q are constants. The gradient of the curve at $(-2, 8)$ and $(0, 6)$ are -7 and 5 respectively. Find

(a) the values of p and q. (*Ans* : $p = 6,\ q = 5$) [3 marks]

(b) the equation of the curve. (*Ans* : $y = 3x^2 + 5x + 6$) [3 marks]
[*klon buku teks*]

Answer :

(a) (b)

⇒ ***second derivative → equation of curve***

24 Given that $\frac{d^2y}{dx^2} = 2x^3 - 4$. When $x = -2$, the values of y and $\frac{dy}{dx}$ are 3 and -3 respectively. Find y in terms of x. (*Ans* : $\frac{1}{10}x^5 - 2x^2 - 19x - \frac{119}{5}$)
[4 marks] [*Forecast*]

Answer :

25 Given that $\frac{d}{dx}\left(\frac{dy}{dx}\right) = x + 3$, $\frac{dy}{dx} = 5$ and $y = -11$ when $x = -2$.

Find y in terms of x. (*Ans* : $\frac{1}{6}x^3 + \frac{3}{2}x^2 + 9x + \frac{7}{3}$)
[4 marks] [*Forecast*]

Answer :

MIND think :

$$y = f(x) \underset{\text{integral}}{\overset{\text{first derivative}}{\rightleftarrows}} \frac{dy}{dx} = f'(x) \underset{\text{integral}}{\overset{\text{second derivative}}{\rightleftarrows}} \frac{d^2y}{dx^2} = f''($$

⇒ gradient of tangent / normal @ equation of tangent / normal → equation of curve

26 It is given that the gradient of a normal to a curve at any point (x, y) is $\frac{1}{6x-2}$. If the curve passes through point $(2, 2)$, find

(a) the gradient function of the curve,

(b) the equation of the curve. (Ans : $y = 10 + 2x - 3x^2$)

[4 marks] [*clon textbook*]

Answer :

(a) (b)

27 The gradient of a curve at any point (x, y) is given by $\frac{dy}{dx} = x^2 + k$, where k is a constant. If the tangent of the curve at the point $(3, 5)$ is parallel to the line $11x - y + 5 = 0$, find

(a) the value of k, (Ans : 2) [2 marks]

(b) the equation of the curve. (Ans : $y = \frac{1}{3}x^3 + 2x - 10$) [3 marks]

[*Forecast*]

Answer :

(a) (b)

MIND think :

- tangent $\rightarrow m_1$
- tangent parallel to x-axis $\rightarrow \frac{dy}{dx}$ @ $m_1 = 0$
- tangent parallel to a line $\rightarrow$ line : m_1
- tangent perpendicular to a line $\rightarrow$ line : m_2
- normal $\rightarrow m_2$
- normal parallel to a line $\rightarrow$ line : m_2
- normal perpendicular to a line $\rightarrow$ line : m_1

$m_1 \times m_2 = -1$

28 The gradient function of a curve is $\frac{dy}{dx} = \frac{1}{(3-x)^2} + k$, where k is a constant. Given that the tangent to the curve at the point $(2, -5)$ is parallel to x-axis. Find

(a) the value of k, (*Ans* : -1) [2 marks]

(b) the equation of the curve. (*Ans* : $y = \frac{1}{3-x} - x - 4$) [3 marks]

[*Forecast*]

Answer :

(a) (b)

29 A curve has a gradient function of $4x^3 - px$, where p is a constant. The tangent to the curve at point (2, 5) is perpendicular to the line $x + 8y = 1$. Find

(a) the value of p, (*Ans* : 12) [3 marks]

(b) the equation of the curve. (*Ans* : $y = x^4 - 6x^2 + 13$) [3 marks]

[*Forecast*]

Answer :

(a) (b)

30 A curve has a gradient function, $\frac{dy}{dx} = 2x + k$, where k is a constant. The gradient of the normal to the curve at point $(1, 5)$ is $\frac{2}{3}$. Find

(a) the value of k, (*Ans* : $-\frac{7}{2}$) [2 marks]

(b) the equation of the curve. (*Ans* : $y = x^2 - \frac{7}{2}x + \frac{15}{2}$) [3 marks]

[*Forecast*]

Answer :

(a) (b)

31 The gradient of a curve is given by $f'(x) = 2x - 1$. If the normal of the curve at the point $P(h, k)$ is $y = 3x + 2$, find

(a) the coordinates of P, [*Ans* : $(\frac{1}{3}, 3)$] [2 marks]

(b) the equation of the curve. (*Ans* : $y = x^2 - x + \frac{29}{9}$) [3 marks]

[*Forecast*]

Answer :

(a) (b)

⇒ ***turning point → equation of curve***

32 The gradient of function of a curve is $\frac{dy}{dx} = kx - 6$, where k is a constant. It is given that the curve has a turning point at (2, 1). Find

(a) the value of k, (*Ans* : 3)

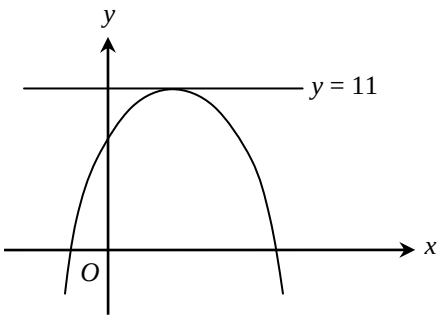
(b) the equation of the curve. (*Ans* : $y = \frac{3}{2}x^2 - 6x + 7$)

[4 marks] [*2009, No.19*]

Answer :

(a) (b)

33 The diagram shows the curve $y = g(x)$. The straight line is a tangent to the curve.



Given $g'(x) = -4x + 8$, find the equation of the curve. (*Ans* : $-2x^2 + 8x + 3$)

[4 marks] [*2018, No.7*]

Answer :

MIND think :

- turning point → $\frac{dy}{dx} = 0$

⇒ ***problems solving***

34 A container is fully filled with water of 10 cm high. Due to the leakage of the container, after t second, the rate of change of the height of the water in the container is given by $20\frac{dh}{dt}+5+16t=0$. Find the height of the water after 2 seconds. (*Ans* : 7.9)

[3 marks] [*Forecast*]

Answer :

35 James is painting a wall at a rate of $(25 - t)$ m^2 per hour, where t is the number hours after he started painting. Given the area of the wall is 132 m^2, find the time, in hours, will it take for him to complete painting the wall in a day. (*Ans* : 6)

[4 marks]
[*Forecast*]

Answer :

36 Stasha is a small entrepreneur, he want to invest RM2000 in a spice business. She is confident that her investment, RMy, will increase at a rate $2(x + 121)$, which x is the time in year. Find the number of year it takes for her investment money to be doubled from her **original** investment. (*Ans* : 8)

[4 marks]
[*Forecast*]

Answer :

==========

3.3 ***Definite Integral***

3.3.1 Determine the value of definite integral for algebraic functions.

==========

37 Find the value of each of the following :

(a) $\int_1^2 x^2 - 2 \ dx$ (*Ans* : $\frac{1}{3}$)

(b) $\int_4^9 \sqrt{x} - \frac{1}{\sqrt{x}} \ dx$ (*Ans* : $\frac{32}{3}$)

Answer :

(a) (b)

38 Find the value of each of the following :

(a) $\int_{-2}^1 x\,(x^2 - 5)\,dx$. (*Ans* : $3\frac{3}{4}$)

(b) $\int_1^4 \frac{3x + 1}{\sqrt{x}}\,dx$ (*Ans* : 16)

Answer :

(a) (b)

MIND think :

$\int_a^b f(x)\ dx = \left[F(x) \right]_a^b = F(b) - F(a)$ $\qquad$ $\int_a^a f(x)\ dx = 0$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ @ $\int_a^b f(x)\,dx = k \rightarrow \int_b^a f(x)\,dx = -k$

$\int_a^b k\ f(x)\,dx = k \int_a^b f(x)\,dx$, k is a constant $\qquad$ $\int_a^b [f(x) + g(x)]\ dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$, where $a < b < c$

39 Find the value of each of the following :

(a) $\int_0^1 \frac{(2t-4)^5}{2-t}\,dt$. (*Ans* : -198.4)

(b) $\int_0^1 \frac{x^2+2x}{(1+x)^2}\,dx$. (*Ans* : $\frac{1}{2}$)

Answer :

(a) (b)

⇒ ***definite integrals ~ 1***

40 Find $\int_4^a (x+1)\,dx$, in terms of a. (*Ans* : $\frac{a^2}{2}+a-12$)

[3 marks] [*2011, No.21*]

Answer :

41 A force , F Newton , acting on a particle at a distance of x meters , from a fixed point O with the relationship $F = 3x + 2x^2$.

Find the work, in *Nm* (joule), which is done when the particle was moved from the position x = 2 m to x = 5 m. (Work = $\int F\,dx$) (*Ans* : $109\frac{1}{2}$)

[3 marks] [*Forecast*]

Answer :

⇒ ***definite integrals ~ 2***

42 Given that $\int_{-1}^{k} (2x-3)\ dx = 6$, where $k > -1$, find the value of k. (*Ans* : 5)

[4 marks] [*2004, No.22*]

Answer :

43 Given $\int_{1}^{h} (2x-6)\ dx = -4$, find the value of h. (*Ans* : 3)

[3 marks] [*2016, No.5*]

Answer :

44 Given that $\frac{dy}{dx} = 6$ and $\int_{0}^{4} y\ dx = 12$. Find y in term of x. (*Ans* : $y = 6x - 9$)

[3 marks] [*Forecast*]

Answer :

⇒ ***definite integrals ~ 3***

45 Given that $\int_{2}^{7} h(x)\ dx = 3$, find

(a) $\int_{7}^{2} h(x)\ dx$, (*Ans* : -3)

(b) $\int_{2}^{7} [5 - h(x)]\ dx$. (*Ans* : 22)

[4 marks] [***2007, No.21***]

Answer :

(a) (b)

46 Given $\int_1^3 f(x)\ dx = 4$, find

(a) $\int_3^1 2f(x)\ dx$,

(b) $\int_1^3 [1 + f(x)]\ dx$ (*Ans* : 6)

[4 marks] [*2014, No.19*]

Answer :

(a) (b)

⇒ ***definite integrals ~ 4***

47 Given that $\int_2^6 f(x)\ dx = 7$ and $\int_2^6 [2f(x) - kx]\ dx = 10$, find the value of k. (*Ans* : $\frac{1}{4}$)

[4 marks] [*2005, No.21*]

Answer :

48 Given that $\int_5^m f(x)\ dx = 6$ and $\int_5^m [f(x) + 2]\ dx = 14$, find the value of m. (*Ans* : 9)

[3 marks] [*2009, No.18*]

Answer :

⇒ ***definite integrals ~ 5***

49 Given that $\int_1^5 g(x)\ dx = 8$, find

(a) the value of $\int_5^1 g(x)\ dx$, (*Ans* : −8)

(b) the value of k if $\int_1^5 [kx - g(x)]\ dx = 10$. (*Ans* : $\frac{3}{2}$)

[4 marks] [***2006, No.21***]

Answer :

(a) (b)

50 Given that $\int_2^7 f(x)\ dx = 10$, find

(a) the value of $\int_7^2 f(x)\ dx$,

(b) the value of k if $\int_2^7 [\ f(x) - k\ ]\ dx = 25.$ (*Ans* : -3)

[3 marks] [*2012, No.21*]

Answer :

(a) (b)

51 Given $\int_3^k f(y)\ dy = \frac{3}{2}$, find

(a) $\int_k^3 4f(y)\ dy$

(b) the value of k such that $\int_3^k [2 - f(y)]\ dy = 4.$ (*Ans* : $\frac{23}{4}$)

[4 marks] [*2019, No.18*]

Answer :

(a) (b)

⇒ ***definite integrals ~ 6***

52 Given $\int_0^1 f(x)\ dx = 3$ and $\int_1^4 f(x)\ dx = 5$. Find $\int_4^0 [\ 2f(x) - x\ ]\ dx$. (*Ans* : -8)

[3 marks] [*Forecast*]

Answer :

53 Given that $\int_{-2}^{1} h(x)\,dx = \int_{1}^{3} h(x)\,dx = 6$, find the value of $\int_{-2}^{3} [\,3h(x) - 2\,]\,dx$. (*Ans* : 26)

[3 marks] [*Forecast*]

Answer :

54 Given that $\int_{-2}^{3} f(x)\,dx = 5$, $\int_{-2}^{1} f(x)\,dx = 2$, and $\int_{3}^{1} g(x)\,dx = k$, find the value of k such that $\int_{1}^{3} [\,kf(x) + g(x) - kx\,]\,dx = 6$. (*Ans* : −3)

[3 marks] [*Forecast*]

Answer :

⇒ ***definite integrals*** *~ 7*

55 Given that $\int_{1}^{3} f(x)\,dx = 7$, find the value of $\int_{1}^{2} f(x)\,dx + \int_{2}^{3} [\,f(x) - 5\,]\,dx$. (*Ans* : 2)

[3 marks] [*Forecast*]

Answer :

56 Given $\int_{1}^{4} f(x)\,dx = 5$, find the value of $\int_{1}^{2} 3f(x)\,dx - \int_{4}^{2} [\,3f(x) + 5x\,]\,dx$. (*Ans* : 45)

[3 marks] [*Forecast*]

Answer :

⇒ ***definite integrals ~ 8 - integrals by reversing differentiation***

57 Given $\frac{d}{dx}\left(\frac{2x}{3-x}\right) = g(x)$, find $\int_1^2 g(x)\ dx$. (*Ans* : 3)

[3 marks] [*2010, No.19*]

Answer :

58 Given $y = \frac{5x}{x^2+1}$ and $\frac{dy}{dx} = g(x)$, find the value of $\int_0^3 2\,g(x)\ dx$. (*Ans* : 3)

[3 marks] [*2011, No.19*]

Answer :

59 Given that $y = \sin x$ and $\frac{dy}{dx} = \cos x$, find the value of $\int_0^{\frac{1}{2}\pi} \frac{2}{3}\cos x\ dx$. (*Ans* : $\frac{2}{3}$)

[3 marks] [*Forecast*]

Answer :

60 Given that $\frac{d}{dx} f(x) = g(x)$, $\int_1^3 2\,g(x)\,dx = 4$, and $f(1) = 1$. Find the value of $f(3)$. (*Ans* : 3)

[3 marks] [*Forecast*]

Answer :

61 Given that $y = \frac{1}{8}(2x-1)^4$ and $\frac{dy}{dx} = p(x)$, find the value of $\int_1^2 \frac{1}{2} p(x)\, dx$. (*Ans* : 5)

[3 marks] [*Forecast*]

Answer :

62 (a) Show that $\frac{d}{dx}\left(\frac{3x^2-4\sqrt{x}}{x}\right) = \frac{3x^2+2\sqrt{x}}{x^2}$. [2 marks]

(b) Hence, find the value of $\int_1^9 \frac{3x^2+2\sqrt{x}}{2x^2}\, dx$. (*Ans* : $\frac{40}{3}$) [2 marks]

[*Forecast*]

Answer :

(a) (b)

==========

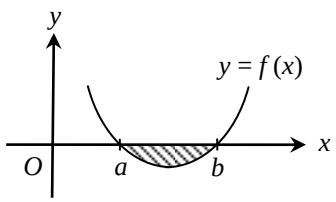
3.3.2 Investigate and explain the relation between the limit of the sum of areas of rectangles and the area under a curve.

3.3.3 Determine the area of a region.

==========

⇒ ***area 1***

63 The diagram shows the curve $y = f(x)$ cutting the x-axis at $x = a$ and $x = b$.

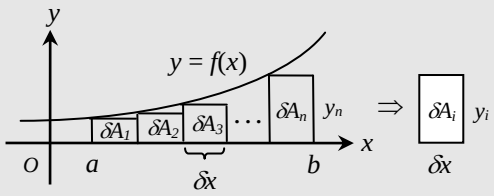


Given that the area of shade region is 5 unit^2, find the value of $\int_a^b 2f(x)\, dx$. (*Ans* : −10)

[2 marks] [***2006, No.20***]

Answer :

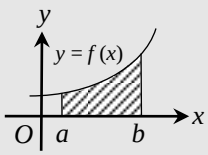
MIND think :



area of n rectangle strips / area under the curve

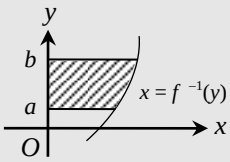
$$= \sum_{i=1}^{n} y_i \, \delta x = \lim_{\delta x \to 0} \sum_{i=1}^{n} y_i \, \delta x = \int_a^b y \, dx$$

$$\delta x = \frac{b-a}{n} \sim n \to \infty, \quad \delta x \to 0$$



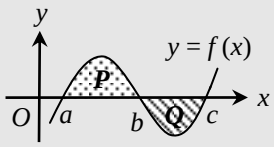
area of shaded region on x-axis

$$= \int_a^b y \, dx = \int_a^b f(x) \, dx$$

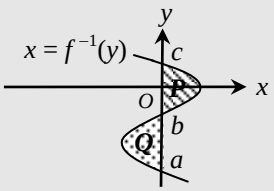


area of shaded region on y-axis

$$= \int_a^b x \, dy = \int_a^b f^{-1}(y) \, dy$$

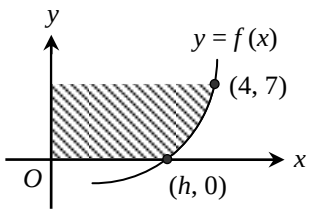


- $\int_a^b f(x)\,dx = h \Rightarrow$ area $\boldsymbol{P} \overset{(+)}{=} h$
- $\int_b^c f(x)\,dx = k \Rightarrow$ area $\boldsymbol{Q} = |k|$



- $\int_b^c f^{-1}(y)\,dy = h \Rightarrow$ area $\boldsymbol{P} \overset{(+)}{=} h$
- $\int_a^b f^{-1}(y)\,dy = k \Rightarrow$ area $\boldsymbol{Q} = |k$

64 The diagram shows part of the curve $y = f(x)$ which passes through the point $(h, 0)$ and $(4, 7)$

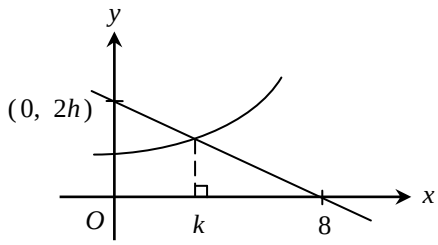


Given that the area of the shaded region is 22 unit^2, find the value of $\int_h^4 f(x)\,dx$. (*Ans* : 6)

[4 marks] [*2009, No.21*]

Answer :

65 The diagram shows a graph of a curve $y = g(x)$ and a straight line $y = f(x)$.



Given $\int_0^k g(x)\,dx + \int_k^8 f(x)\,dx = 10$.

(a) On the diagram above, shade the region represented by $\int_0^k g(x)\,dx + \int_k^8 f(x)\,dx$.

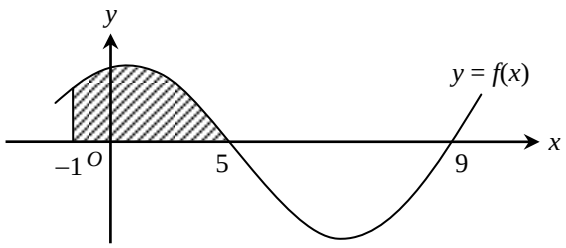
(b) Find the area, in terms of h, for the region bounded by y-axis, $y = g(x)$ and $y = f(x)$. (*Ans* : $8h - 10$)

[4 marks] [*2015, No.17*]

Answer :

(b)

66 The diagram shows part of the curve $y = f(x)$.



The shaded region is defined as $\int_a^b f(x)\ dx = 4$.

(a) State the value of a and of b.

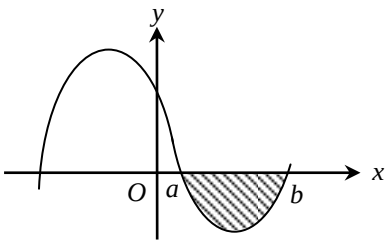
(b) It is given that the area bounded by $y = f(x)$ and the x-axis from $x = -1$ to $x = 9$ is 12. State the value of $\int_5^9 f(x)\ dx$.

[2 marks] [*2017, No.2*]

Answer :

(a) (b)

67 The diagram shows a shaded region bounded by the graph $y = g(x)$ and x-axis from $x = a$ to $x = b$.



(a) It is given that $\frac{d}{dx}[2h(x)] = g(x)$ and $\left[h(x)\right]_a^b = -7$. State the area of the shaded region.

(b) The graph $y = g(x)$ passes through point (3, 19). Given $\frac{d}{dx}[g(x)] = 3x^2 + 2x - 6$, find $g(x)$ in terms of x.

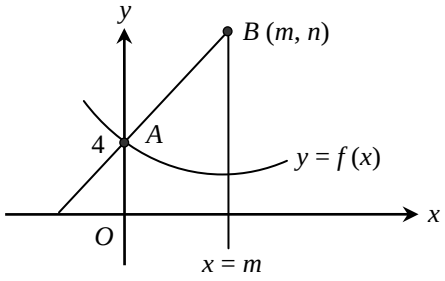
[*Ans* : $g(x) = x^3 + x^2 - 6x + 1$]

[4 marks] [*2019, No.19*]

Answer :

(a) (b)

68 The diagram show part of curve $y = f(x)$, which passes through y-axis at A.



(a) The straight line AB has a gradient of 3. Express n in terms of m. (*Ans* : $n = 3m + 4$)

(b) Given $\int_0^m f(x)\,dx = 12$ and the area bounded by straight line AB, $y = f(x)$ and $x = m$ is 66 unit^2. Find the value of m. (*Ans* : 6)

[4 marks] [*Forecast*]

Answer :

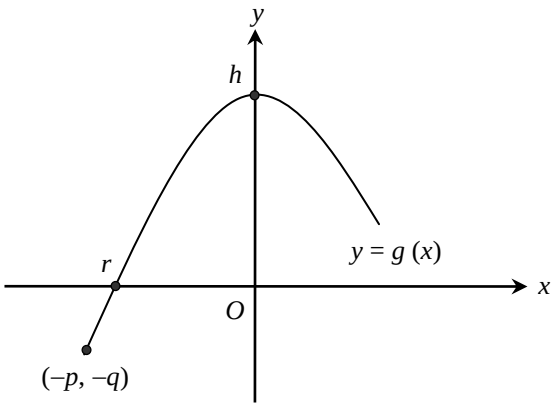
(a) (b)

69 The diagram in the answer space shows part of the curve $y = g(x)$. On the diagram in the answer space, shade the region that represented by the following :

$$\int_{-q}^{h} -p\,dy - \left[\left|\int_{-p}^{r} g(x)\,dx\right| - \int_{0}^{r} g(x)\,dx\right]$$

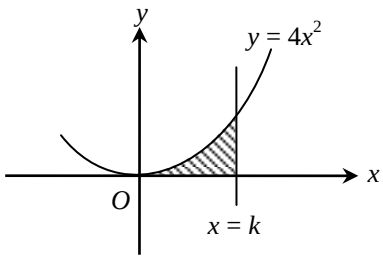
[2marks] [*Forecast*]

Answer :



⇒ ***area 2***

70 The diagram shows the curve $y = 4x^2$ and the straight line $x = k$.

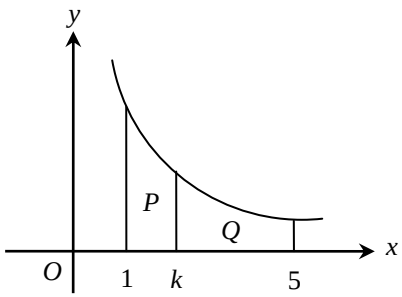


If the area of the shaded region is 36 unit^2, find the value of k. (*Ans* : 3)

[3 marks] [***2003, No.18***]

Answer :

71 The diagram shows a curve $y = \frac{5}{x^2}$.

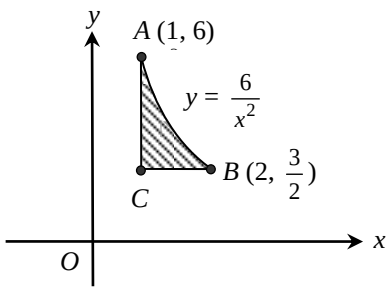


Given the area of region P is equal to the area of region Q. Find the value of k. (*Ans* : $\frac{5}{3}$)

[3 marks] [***Forecast***]

Answer :

72 The diagram shows part of the curve $y = \frac{6}{x^2}$ and the straight lines $x = 1$ and $y = \frac{3}{2}$.

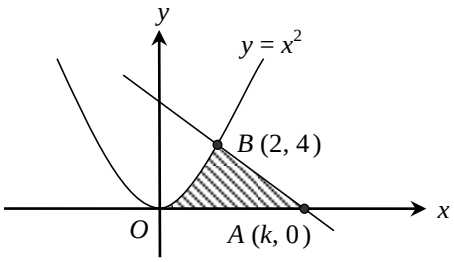


Find the area, in units^2, of the shaded region ABC. ($Ans: \frac{3}{2}$)

[4 marks] [***Forecast***]

Answer :

73 In the diagram, the straight line AB intersects the curve $y = x^2$ at point $B(2, 4)$.

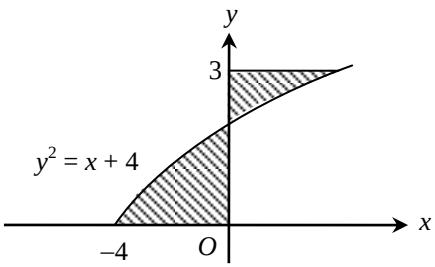


Given that the area of shaded region is $10\frac{2}{3}$ units^2. Find the value of k. (Ans : 6)

[4 marks] [***Forecast***]

Answer :

74 The diagram shows part of a curve, $y^2 = x + 4$.

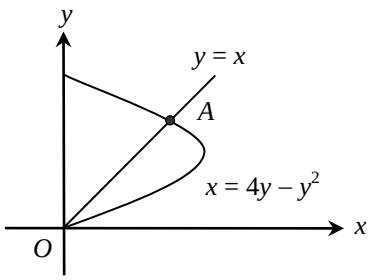


Find the area, in units^2, of the shaded region. (*Ans* : $\frac{23}{3}$)

[4 marks] [***Forecast***]

Answer :

75 The diagram shows the curve $x = 4y - y^2$ intersecting the straight line $y = x$ at point A.



Find the area, in units^2, bounded by the curve $x = 4y - y^2$ and the straight line $y = x$. (*Ans* : $\frac{9}{2}$)

[4 marks] [***Forecast***]

Answer :

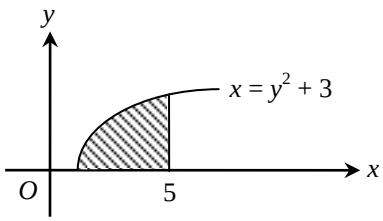
==========

3.3.3 Investigate and explain the relation between the limits of the sum of volumes of cylinders and the generated volume by revolving a region.

3.3.5 Determine the generated volume of a region revolved at the *x*-axis or the *y*-axis.

==========

76 The diagram shows the the graphs $x = y^2 + 3$ and $x = 5$.

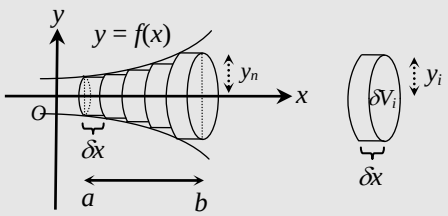


Find the volume generated, in term of π, when the shaded region is rotated through $360°$ about the x-axis. (*Ans* : 2π)

[3 marks] [***Forecast***]

Answer :

MIND think :



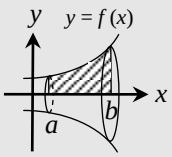
volumes of n cylinders / the generated volume

$$= \sum_{i=1}^{n} \pi (y_i)^2 \, \delta x = \lim_{\delta x \to 0} \sum_{i=1}^{n} \pi (y_i)^2 \, \delta x = \pi \int_a^b y^2 \, dx$$

$$\delta x = \frac{b-a}{n} \sim n \to \infty, \quad \delta x \to 0$$

volume of shaded region revolved $360°$ about x-axis

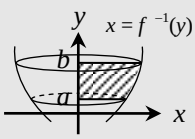
$$\pi \int_a^b y^2 \, dx$$



revolved

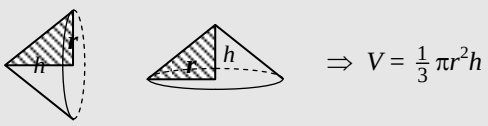
90° [$\times \frac{1}{4}$]

180° [$\times \frac{1}{2}$]

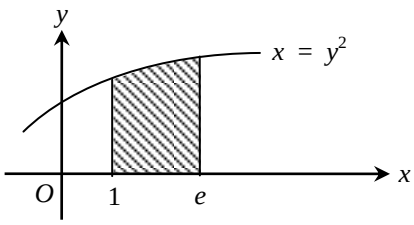


volume of shaded re revolved $360°$ about y-axis

$$\pi \int_a^b x^2 \, dy$$



77 The diagram shows part of the curve $x = y^2$.

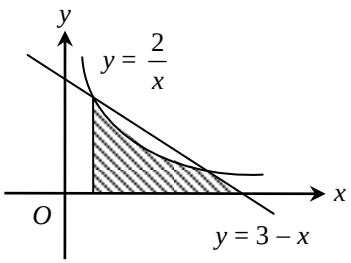


When the region bounded by the curve $x = y^2$, the straight lines $x = 1$ and $x = e$, and the x-axis is revolved completely about the x-axis, the volume generated is $\frac{3\pi}{2}$ unit^3. Find the value of e. (*Ans* : 2)

[3 marks]
[*Forecast*]

Answer :

78 The diagram shows the curve $y = \frac{2}{x}$ and the straight line $y = 3 - x$.

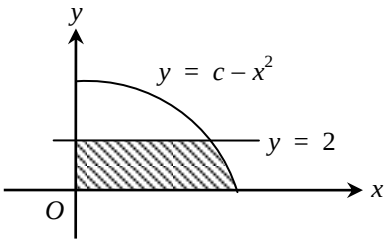


Find the volume generated, in term of π, when the shaded region is rotated through 360° about the x-axis. (*Ans* : $\frac{7}{3}\pi$)

[4 marks] [*Forecast*]

Answer :

79 The shaded region in the diagram is revolved $180°$ about the y-axis.

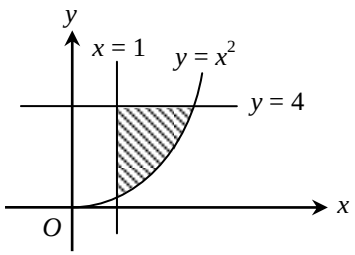


If the volume generated is 3π unit^3, find the value of c. (*Ans* : 4)

[3 marks] [*Forecast*]

Answer :

80 The diagram shows the curve $y = x^2$, straight line $x = 1$ and straight line $y = 4$.



Find the volume generated, in term of π, when the shaded region is rotated through $360°$ about the y-axis. (*Ans* : $\frac{9}{2}\pi$)

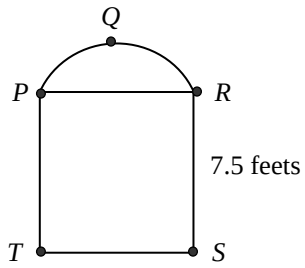
[4 marks] [*Forecast*]

Answer :

3.4 Application of Integration

3.4.1 Solve problems involving integration. ~ area and volume

81 The diagram shows the front view of a door.



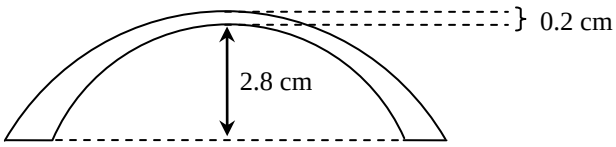
PQR is part of the graph $y = 9 - \frac{x^2}{6}$. Q is the highest point from TS. Find the surface area of the door.

(*Ans* : 51)

[4 marks] [*forecast*]

Answer :

82 The diagram shows the shape of the cross-section of a machine cover produced by a 3D printer.



The cover is made from a kind of plastic. The internal and the external shapes of the cover are represented by the equations $y = 2.8 - \frac{1}{15}x^2$ and $y = k - \frac{1}{20}x^2$ respectively.

(a) State the value of k.

(b) If the cost of 1 cm^3 of the plastic is 8 cents, estimate the cost, in RM, of the plastic used to make the same 20 covers. [Use $\pi = 3.142$]

(*Ans* : 156.85)

[4 marks] [*clon textbook*]

Answer :

(a)

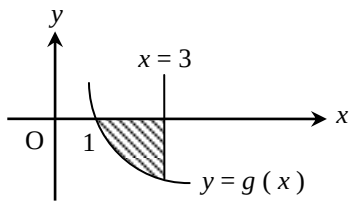
(b)

PAPER 2

⇒ ***Part A → 6 – 8 marks***

~ ***area 1 ; equation of curve,***

83 The diagram shows the shaded region bounded by the curve $y = g(x)$, the x-axis and the straight line $x = 3$.



It is given that the area of the shaded region is 10 unit^2.

(a) Find

(i) $\int_1^3 g(x)\ dx$, (*Ans* : −10)

(ii) $\int_1^3 [\, x - 2\, g(x)\,]\, dx$, (*Ans* : 24)

[4 marks]

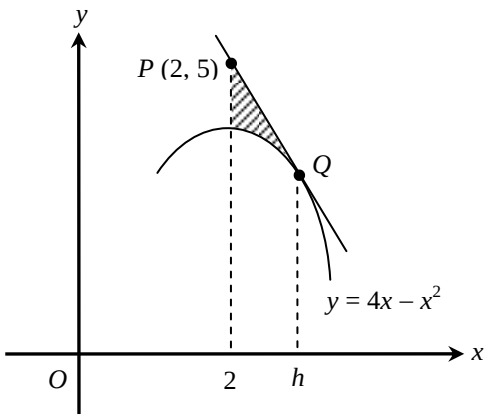
(b) Given $g'(x) = 2x - 8$, find $g(x)$ in terms of x (*Ans* : $g(x) = x^2 - 8x + 7$) [3 marks]

[***2010, No.4***]

Answer :

~ *area 2*

84 The diagram shows the curve $y = 4x - x^2$ and tangent to the curve at point Q passes point P.



(a) Show that $h = 3$. [4 marks]

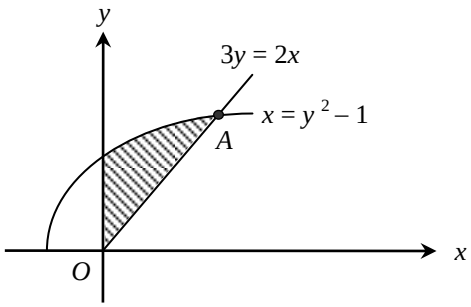
(b) Calculate the area of the shaded region. (*Ans* : $\frac{1}{3}$) [4 marks]

[***2019, No.4***]

Answer :

~ *volume*

85 The diagram shows a curve $x = y^2 - 1$ intersects the straight line $3y = 2x$ at point A.



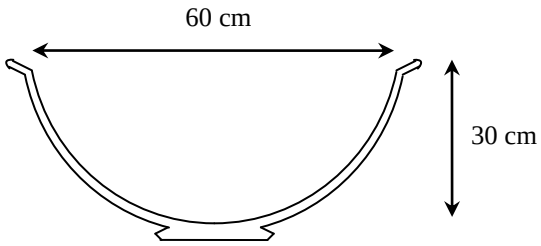
Calculate the volume generated when the shaded region is revolved $360°$ about y-axis. (*Ans* : $\frac{52}{15}\pi$)

[6 marks]

[***2003, No.9(b)***]

Answer :

86 The diagram shows a side elevation of the inner surface of a bowl which can be represented by the equation $y = ax^2$.



(a) Show that $a = \frac{1}{30}$. [2 marks]

(b) Determine the volume, in cm^3, of water needed to fill the bowl to a depth of 20 cm.

(*Ans* : 6000π)

[4 marks]
[***2014, No.6***]

Answer :

⇒ Part A → 6 – 8 marks

~ differentiation + integration : second derivative ; equation of curve

87 (a) Given that $\frac{dy}{dx} = 2x + 2$ and $y = 6$ when $x = -1$, find y in terms of x.
(*Ans* : $y = x^2 + 2x + 7$ [3 marks]

(b) Hence, find the values of x if $x^2 \frac{d^2y}{dx^2} + (x - 1)\frac{dy}{dx} + y = 8$. (*Ans* : -1, $\frac{3}{5}$) [4 marks]

[***2003, No.3***]

Answer :

~ differentiation + integration : equation of tangent, equation of normal ; equation of curve

88 A curve has a gradient function of $px^2 - 4x$, where p is a constant. The tangent of the curve at the point (1, 3) is parallel to the straight line $y + x - 5 = 0$. Find

(a) the value of p, (*Ans* : 3) [3 marks]

(b) the equation of the curve. (*Ans* : $y = x^3 - 2x^2 + 4$) [3 marks]

[***2005, No.2***]

Answer :

~ *differentiation + integration : turning point ; equation of curve*

89 The gradient function of a curve which passes through A $(1, -12)$ is $3x^2 - 6x$. Find

(a) the equation of the curve, (Ans : $y = x^3 - 3x^2 - 10$) [3 marks]

(b) the coordinates of the turning points of the curve and determine whether each of the turning points is a maximum or a minimum. [Ans : $(0, -10)$ maximum, $(2, -14)$ minimum] [5 marks]

[***2004, No.5***]

Answer :

90 A curve with gradient function $2x - \frac{2}{x^2}$ has a turning point at $(k, 8)$.

(a) Find the value of k. (Ans : 1) [3 marks]

(b) Determine whether the turning point is a maximum or a minimum point.

(Ans : minimum) [2 marks]

(c) Find the equation of the curve. (Ans : $y = x^2 + \frac{2}{x} + 5$) [3 marks]

[***2007, No.4***]

Answer :

91 The gradient function of a curve is $hx^2 - kx$, where h and k are constants. The curve has a turning point at $(3, -4)$. The gradient of the tangent to the curve at the point $x = -1$ is 8. Find

(a) the value of h and of k, (*Ans* : $h = 2$, $k = 6$) [5 marks]

(b) the equation of the curve. (*Ans* : $y = \frac{2}{3}x^3 - 3x^2 + 5$) [3 marks]

[***2009, No.3***]

Answer :

92 A curve has a gradient function $kx - 6$, where k is a constant. Given that the minimum point of the curve is $(3, 5)$, find

(a) the value of k, (*Ans* : 2) [3 marks]

(b) the y–intercept of the curve (*Ans* : 14) [3 marks]

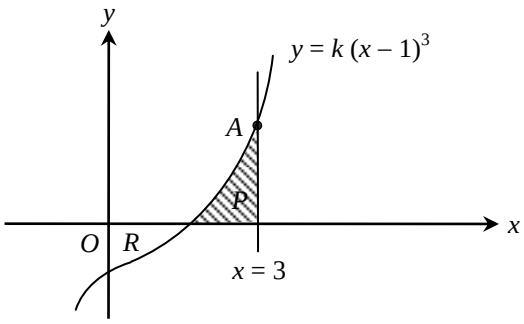
[***2012, No.3***]

Answer :

⇒ Part B → 10 marks

~ differentiation + integration : gradient of tangent, gradient of normal - 1 ; area, volume

93 The diagram shows part of the curve $y = k\,(x - 1)^3$, where k is a constant.



The curve intersects the straight line $x = 3$ at point A. At point A, $\frac{dy}{dx} = 24$.

(a) Find the value of k. (*Ans* : 2) [3 marks]

(b) Hence, calculate

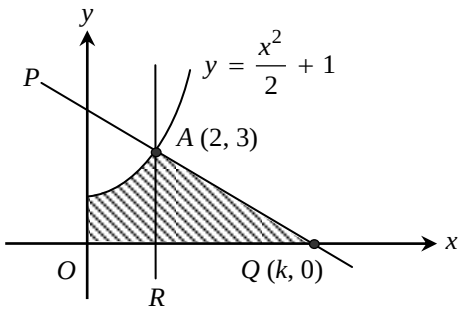
(i) the area of the shaded region P,
(*Ans* : 8)

(ii) the volume generated, in terms of π, when the shaded region R which is bounded by the curve, the x-axis and the y-axis, is revolved through 360° about the x-axis. (*Ans* : $\frac{4}{7}\pi$)

[7 marks]
[***2007, No.10***]

Answer :

94 In the diagram, the straight line PQ is normal to the curve $y = \frac{x^2}{2} + 1$ at $A\,(2, 3)$. The straight line AR is parallel to the y-axis.



Find

(a) the value of k, (*Ans* : 8) [3 marks]

(b) the area of the shaded region, (*Ans* : $12\frac{1}{3}$) [4 marks]

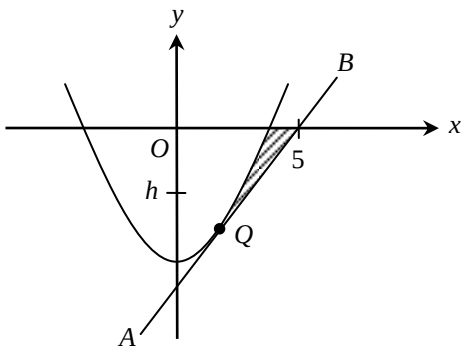
(c) the volume generated, in term of π, when the region bounded by the curve, the y-axis and the straight line $y = 3$ is revolved through 360° about the y-axis. (*Ans* : 4π) [3 marks]

[***2005, No.8***]

Answer :

~ differentiation + integration : gradient of tangent, gradient of normal - 2 ; area, volume

95 The diagram shows a curve $y = 2x^2 - 18$ and straight line AB which is a tangent to the curve at point Q.



It is given that the gradient of the straight line AB is 4.

(a) Find the coordinates of point Q. [*Ans* : $(1, -16)$] [2 marks]

(b) Calculate the area of the shaded region. (*Ans* : $13\frac{1}{3}$) [5 marks]

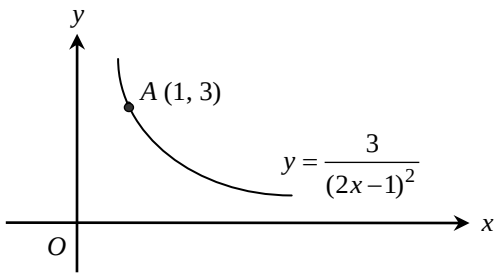
(c) When the region bounded by the curve, the x-axis and the straight line $y = h$ is rotated through 180° about the y-axis, the volume generated is 65π unit^3. Find the value of h. (*Ans* : -10) [3 marks]

[***2017, No.11***]

Answer :

~ differentiation + integration : equation of tangent, equation of normal ; area, volume

96 The diagram shows part of the curve $y = \dfrac{3}{(2x-1)^2}$ which passes through $A\ (1,\ 3)$.



(a) Find the equation of the tangent to the curve at the point A.
($Ans:\ y = -12x + 15$) [4 marks]

(b) A region is bounded by the curve, the x-axis and the straight line $x = 2$ and $x = 3$.

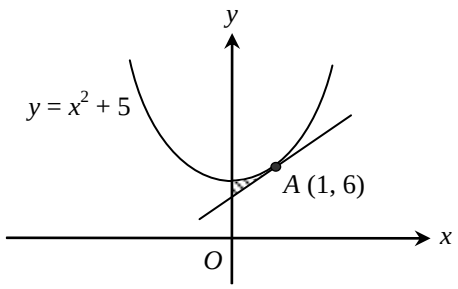
(i) Find the area of the region. ($Ans:\ \frac{1}{5}$)

(ii) The region is revolved through 360° about the x-axis. Find the volume generated, in term of π. ($Ans:\ \frac{49}{1125}\pi$)

[6 marks]
[***2004, No.10***]

Answer :

97 The diagram shows the curve $y = x^2 + 5$ and the tangent to the curve at the point A (1, 6).



Calculate

(a) the equation of the tangent at A, (*Ans* : $y = 2x + 4$) [3 marks]

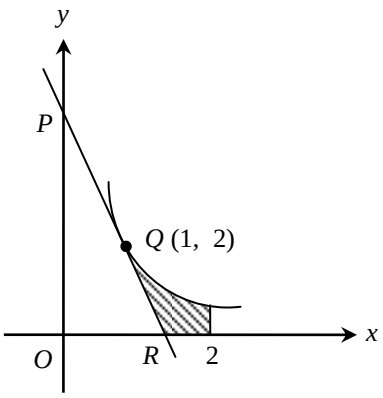
(b) the area of the shaded region, (*Ans* : $\frac{1}{3}$) [4 marks]

(c) the volume of revolution, in terms of π, when the region bounded by the curve and the straight line $y = 7$ is rotated through 360° about the y-axis. (*Ans* : 2π) [3 marks]

[***2008, No.7***]

Answer :

98 The diagram shows the curve $y = \dfrac{8}{(3x-1)^2}$. The straight line PR is tangent to the curve at $Q(1,\ 2)$.



Find

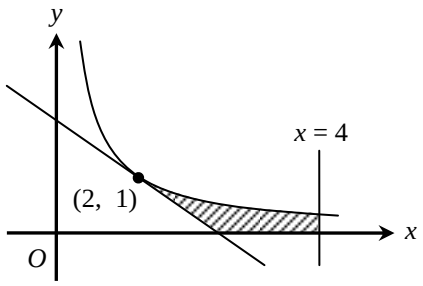
(a) the equation of the straight line PR, (Ans : $y = -6x + 8$) [4 marks]

(b) the area of shaded region. (Ans : $\frac{7}{15}$) [6 marks]

[***2014, No.8***]

Answer :

99 The diagram shows the curve $y = \frac{4}{x^2}$ and the straight line $y = mx + c$. The straight line $y = mx + c$ is a tangent to the curve at $(2,\ 1)$.



(a) Find the value of m and of c. (*Ans* : $m = -1,\ c = 3$) [3 marks]

(b) Calculate the area of the shaded region. (*Ans* : $\frac{1}{2}$) [4 marks]

(c) It is given that the volume of revolution when the region bounded by the curve, the x-axis, the straight lines $x = 2$ and $x = k$ is rotated through $360°$ about the x-axis is $\frac{38\pi}{81}$ unit^3. Find the value of k, such that $k > 2$. (*Ans* : 3) [3 marks]

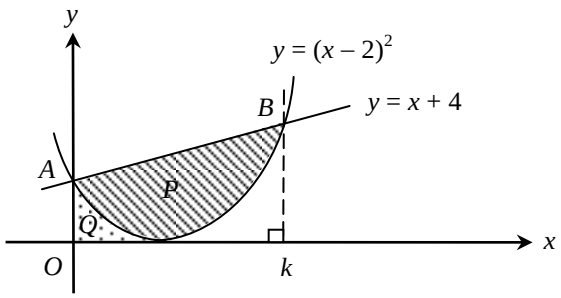
[***2016, No.8***]

Answer :

⇒ ***Part B → 10 marks***

~ ***intersection points - 1 ; area, volume***

100 The diagram shows the straight line $y = x + 4$ intersecting the curve $y = (x - 2)^2$ at the points A and B.



Find

(a) the value of k, (*Ans* : 5) [2 marks]

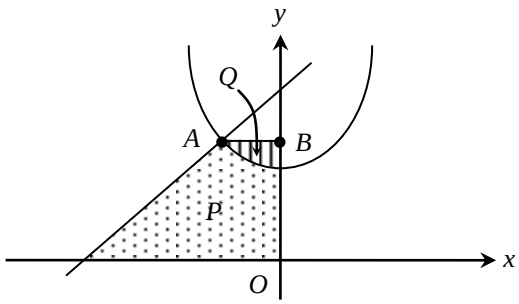
(b) the area of the shaded region P, (*Ans* : $20\frac{5}{6}$) [5 marks]

(c) the volume generated, in term of π, when the shaded region Q is revolved through 360° about the x-axis. (*Ans* : $\frac{32}{5}\pi$) [3 marks]

[***2006, No.8***]

Answer :

101 The diagram shows the straight line $y = x + 6$ intersects the curve $y = \frac{1}{4}x^2 + 3$ at point A.



(a) Find the coordinates of A. [*Ans* : (–2, 4)] [2 marks]

(b) Calculate

(i) the area of the shaded region P, (*Ans* : $\frac{44}{3}$)

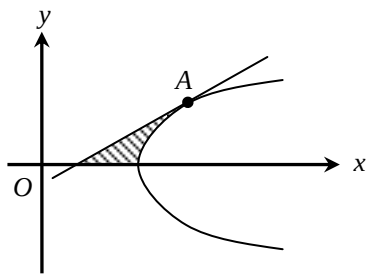
(ii) the volume of revolution, in terms of π, when the shaded region Q is rotated through 360° about the y-axis. (*Ans* : 2π)

[8 marks]

[***2015, No.8***]

Answer :

102 The diagram shows the straight line $4y = x - 2$ touches the curve $x = y^2 + 6$ at point A.



Find

(a) the coordinates of A, [*Ans* : $(10, 2)$] [2 marks]

(b) the area of the shaded region, (*Ans* : $\frac{8}{3}$) [5 marks]

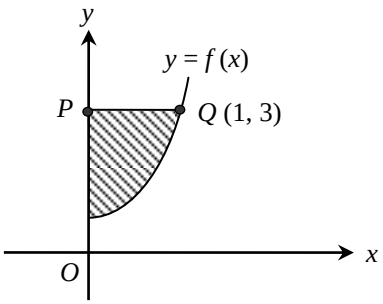
(c) the volume of revolution, in terms of π, when the region bounded by the curve and the straight line $x = 8$ is revolved through $180°$ about the x-axis. (*Ans* : 2π) [3 marks]

[***2018, No.10***]

Answer :

~ *equation of curve ; area, volume*

103 The diagram shows part of the curve $y = f(x)$ which passes through $Q(1, 3)$. The straight line PQ is parallel to the x-axis.



The curve has a gradient function of $2x$. Find

(a) the equation of the curve, ($Ans: y = x^2 + 2$) [3 marks]

(b) the area of the shaded region, ($Ans: \frac{2}{3}$) [4 marks]

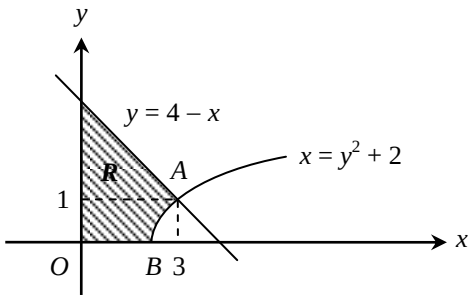
(c) the volume of revolution, in terms of π, when the shaded region is rotated through 360° about y-axis. ($Ans: \frac{1}{2}\pi$) [3 marks]

[***2011, No.8***]

Answer :

~ *area, volume*

104 The diagram shows the curve $x = y^2 + 2$ intersects the straight line $y = 4 - x$ at $A\ (3,\ 1)$ and the x-axis at point B.



Find

(a) the coordinates of point B, [*Ans* : $(2,\ 0)$] [1 mark]

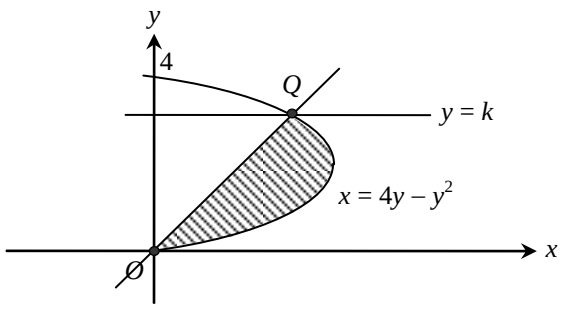
(b) the area of the shaded region $\boldsymbol{R}$, (*Ans* : $\frac{41}{6}$) [6 marks]

(c) the volume generated, in terms of π, when the region bounded by the curve $x = y^2 + 2$, the straight line $x = 3$ and the x-axis is revolved through $360°$ about the x-axis. (*Ans* : $\frac{\pi}{2}$) [3 marks]

[***2013, No.8***]

Answer :

105 The diagram shows the straight line OQ and the straight line $y = k$ intersectin the curve $x = 4y - y^2$ at point Q.



It is given that the area of the shaded region is $\frac{9}{2}$ units.

(a) Find the value of k. (*Ans* : 3) [6 marks]

(b) The region enclosed by the curve and the y-axis is revolved through 360° about the y-axis. Find the volume of revolution, in term of π. (*Ans* : $\frac{512}{15}\pi$) [4 marks]

[***2009, No.7***]

Answer :

FORECAST

⇒ ***Part A → 6 – 8 marks***

~ ***equation of curve***

106 A liquid in the laboratory is heated until the temperature of the liquid becomes 150°C. The liquid is left to cool and at t seconds later, the temperature of the liquid, T°C, decreases at a rate, $\frac{dT}{dt} = -\frac{5}{(t-3)^2}$

(a) Show that $3T = \frac{15}{t-3} + 455$. [4 marks]

(b) Find the time taken for the temperature of the liquid to be half of the initial temperature. (*Ans* : $2\frac{43}{46}$) [2 marks]

Answer :

107 The bacterial population, P, in a culture increases at a rate, $\frac{dP}{dt} = t^2 + 4t + k$, where k is a constant and t is the time in minutes. Initially, $P = 5$ and three minutes later, $P = 569$.

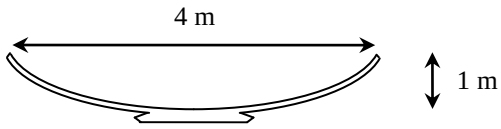
(a) Show that $k = 179$. [4 marks]

(b) Calculate the population, P, for the bacteria when $t = 6$. (*Ans* : 1223) [2 marks]

Answer :

~ integration : area, volume

108 The diagram shows a side view of the inner surface of a parabolic container with a diameter of 4 m and a height of 1 m.



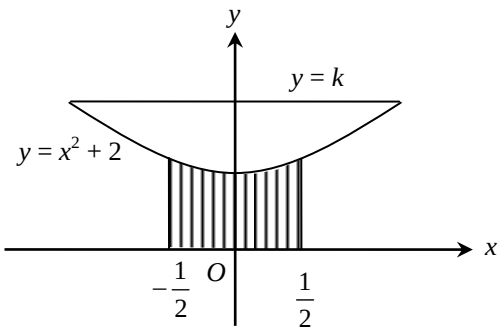
(a) Show that the parabolic equation is given by $y = \frac{1}{4}x^2$. [2 marks]

(b) Find the surface area, in cm^2, of the side view of the container. (*Ans* : $2\frac{2}{3}$) [3 marks]

(c) Find the space, in cm^3, of the container. (*Ans* : 2π) [3 marks]

Answer :

109 Dionysia has a hobby of growing flowers during her free time. The diagram shows the side view of the vase and the vase holder. The shaded area is the vase holder



(a) Calculate the area of the shaded region. (*Ans* : $\frac{25}{12}$) [3 marks]

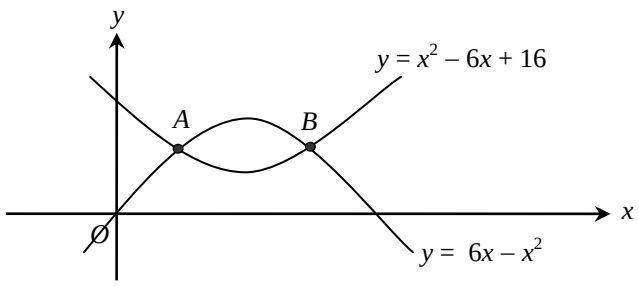
(b) The volume of the vase produced when the region bounded by the curve $y = x^2 + 2$ and the line $y = k$ is rotated through 180° about the y-axis is $\frac{1}{2}\pi$ unit^3, find the value k.

(*Ans* : 3) [4 marks]

Answer :

⇒ ***Part B → 10 marks***
~ ***area, volume***

110 (a) The diagram shows the curve $y = x^2 - 6x + 16$ intersecting the curve $y = 6x - x^2$ at the points A and B.



Find

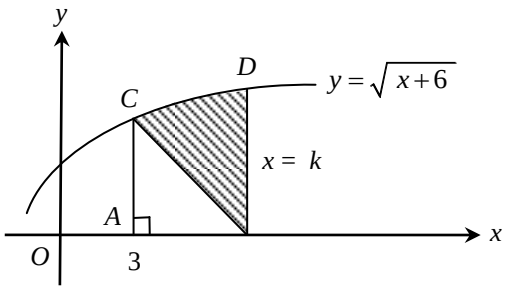
(i) the coordinates of the points A and B, [*Ans* : $A\,(2,\ 8),\ B\,(4,\ 8)$]

(ii) the area bounded by the curve $y = x^2 - 6x + 16$ and the curve $y = 6x - x^2$. (*Ans* : $2\frac{2}{3}$)

[5 marks]

Answer :

(b) The diagram shows part of a curve $y = \sqrt{x+6}$, the straight line $x = k$, and the straight line AC.



When the shaded region is revolved 360° about the x-axis, the volume generated is $42\frac{1}{2}\pi$ unit^3.

Find the value of k. (*Ans* : 8) [5 marks]

Answer :

⇒ Part A → 6 – 8 marks

~ differentiation + integration : integration as inverse of differentiation

111 (a) Differentiate $\frac{x^2}{2x-1}$ with respect to x. [*Ans* : $\frac{2x(x-1)}{(2x-1)^2}$] [3 marks]

(b) Hence, or otherwise, find $\int_0^1 \frac{4x(x-1)}{(2x-1)^2}\,dx$. (*Ans* : 2) [3 marks]

Answer :

~ differentiation + integration : equation of tangent, equation of normal ; equation of curve

112 A curve has a gradient function of $2x + k$, where k is a constant. If the tangent to the curve at the point (3, 6) passes through origin, find

(a) the value of k, (*Ans* : –4) [3 marks]

(b) the equation of the curve. (*Ans* : $y = x^2 - 4x + 9$) [3 marks]

Answer :

113 The slope of a curve at any point (x, y) is given by $kx + 4$, where k is a constant. If $y - 10x + 1 = 0$ is the tangent to the curve at the point $(1, 9)$, find

(a) the value of k, (*Ans* : 6) [3 marks]

(b) the equation of the curve. (*Ans* : $y = 3x^2 + 4x + 2$) [3 marks]

Answer :

114 The second derivative of a curve is given by $\frac{d^2y}{dx^2} = 1 + 2x + x^2$. If the tangent to the curve at the point $(1, -1)$ is $y = x - q$, where q is a constant, find

(a) the value of q, (*Ans* : 2) [2 marks]

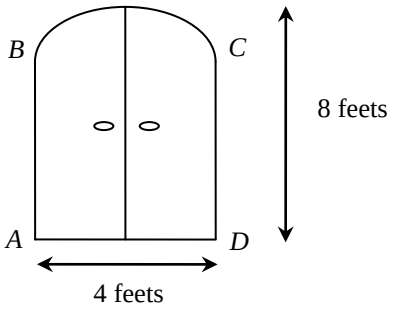
(b) the equation of the curve.

(*Ans* : $y = \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{12} - \frac{4}{3}x - \frac{7}{12}$) [5 marks]

Answer :

~ ***differentiation + integration : turning point ; equation of curve***

115 Walter wanted to make a door with a parabolic top as shown in the diagram.



Given that the gradient function of the curve BC is $px + 2$, where p is a constant. (Assume point A as origin)

(a) Find the equation of the curve BC in the form $y = ax^2 + bx + c$, where a, b and c are constants. (*Ans* : $y = -\frac{1}{2}x^2 + 2x + 6$) [3 marks]

(b) Calculate the cost, in RM, to make the door if the price is RM30 per square feet. (*Ans* : 880) [3 marks]

Answer :

116 Given that $\frac{dy}{dx} = 2x^3 - 16$ and $y = 4\frac{1}{2}$ when $x = -1$, find

(a) the value of x when y is minimum, (*Ans* : 2) [2 marks]

(b) the minimum value of y. (*Ans* : −36) [4 marks]

Answer :

⇒ ***Part B → 10 marks***

~ ***differentiation + integration :***

117 (a) Diagram (a) shows a rectangular piece of manila cardboard $ABCD$. The sides AD and BC are joined to form an open cylinder of height h cm as shown in diagram (b). The perimeter of $ABCD$ is 48 cm and the volume of the cylinder is v cm^3.

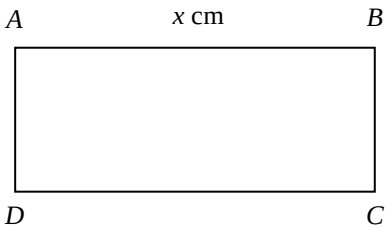


Diagram (a)

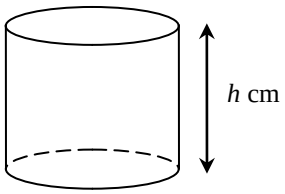


Diagram (b)

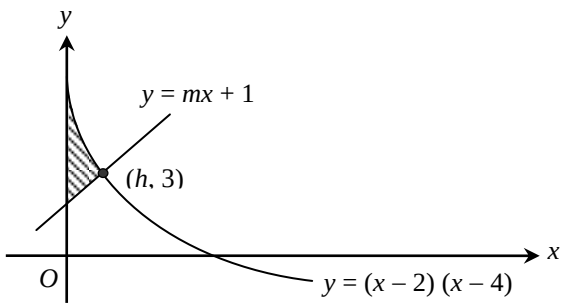
(i) Show that $v = \dfrac{x^2}{4\pi}(24 - x)$.

(ii) Find the value of x and of h such that the volume of the cylinder is a maximum. ($Ans: x = 16, h = 8$)

[5 marks]

Answer :

(b) The diagram shows the straight line $y = mx + 1$ intersecting the curve $y = (x - 2)(x - 4)$ at the point $(h, 3)$.



Find

(i) the value of h and of m, (*Ans* : $h = 1,\ m = 2$)

(ii) the area of the shaded region. (*Ans* : $3\frac{1}{3}$)

[5 marks]

Answer :

PERMUTATIONS AND COMBINATIONS

- ONE PAGE NOTE (OPN)
- WORKSHEET

Dr Ma Chi Nan

FORM 5 TOPIC 4: PERMUTATION AND COMBINATION

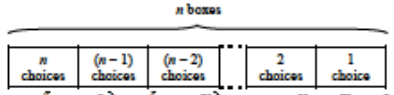
MULTIPLICATION RULE

If an event can occur in m ways and a second event can occur in n ways, both events can occur in $m \times n$ ways

PERMUTATION

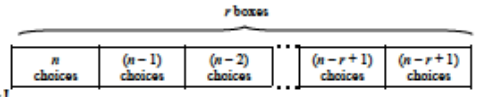
Order of arrangement is important

The number of permutations of n different objects:



$$n! = {}^{n}P_{n} = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$$

The number of permutations of different n objects taking r each time:



$$ {}^{n}P_{r} = \frac{n!}{(n-r)!}$$

where $r \leq n$

COMBINATION

Order of arrangement is not important

The number of combinations of r objects selected from n objects:

$$ {}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!} = \frac{n!}{(n-r)!\,r!}$$

where $r \leq n$.

CIRCULAR PERMUTATION

Clockwise or anticlockwise arrangement are different (eg: round table, …)

The number of circular permutations of n different objects:

$$\frac{n!}{n} = \frac{n(n-1)!}{n} = (n-1)!$$

The number of circular permutations of n different objects taking r each time:

$$\frac{{}^{n}P_{r}}{r}$$

where $r \leq n$.

Clockwise or anticlockwise arrangement are the same (eg: Bracelet, necklace, …)

The number of circular permutations of n different objects:

$$\frac{n!}{2n} \text{ or } \frac{(n-1)!}{2}$$

The number of circular permutations of n different objects taking r each time:

$$\frac{{}^{n}P_{r}}{2r}$$

where $r \leq n$.

IDENTICAL OBJECTS

The number of permutations for n objects involving identical objects:

$$\frac{n!}{a!\,b!\,c!\,\ldots}$$

where $a,b,c,\ldots$are the number of identical objects for each type

WORKSHEET
TOPIC 4 : PERMUTATION AND COMBINATION
[1 – 2 questions → 3 – 7 marks]

==========

4.1 Permutation

4.1.1 Investigate and make generalisation about multiplication rule.

==========

1 Emmanuel has to choose a shirt and a trouser from 6 shirts and 4 trousers in his cupboard. Find the possible number of ways he can do this. (*Ans* : 24)
[2 marks]

Answer :

2 A coin and a dice are tossed simultaneously. Find the number of possible outcomes. (*Ans* : 12)
[2 marks]

Answer :

3 Team Fredie plays four successive football league matches. If the result of each match is either win, lose or draw, find the number of possible outcomes from the four matches. (*Ans* : 81)
[2 marks]

Answer :

4 Find the number of ways a person can guess a 6-digit code to access a cell phone if the digits can be repeated. (*Ans* : 1000000)
[2 marks]

Answer :

5 There are 8 students taking part in a racing event. Find the number of ways that the first three places are to be awarded. (*Ans* : 336)
[2 marks]

Answer :

6 The committee of a company consists of 10 members. A President, a Secretary and a Treasurer are to be elected from the committee. Find the number of ways that these three posts can be filled. (*Ans* : 720)
[2 marks]

Answer :

MIND think :

- The multiplication rule
  ~ If event *A* occur in *r* ways, and event *B* occur in *s* ways ⇒ *the number of ways* = $r \times s$

==========

4.1.2 Determine the number of permutations for

(i) *n* different objects
(ii) *n* different objects taken *r* at a time
(iii) *n* objects involving identical objects

4.1.3 Solve problems involving permutations with certain conditions.

==========

⇒ ***meaning of :*** $n!$, ${}^nP_r$, ${}^nC_r$ (' **!** ' read as factorial)

7 Write the following in term of : $(a)(b)(c) \ldots$; $(a)(b)(c!)$ and $(a)(b!)$, where *a*, *b*, *c* . . . are integers

(a) 8 !

=

=

=

(b) $(n-2)$!

=

=

=

8 Without using calculator, find the value of each of the following : [Use a calculator, check the answer]

(a) $\frac{9!6!}{5!3!}$

=

(b) $\frac{10!-7!}{7!}$

=

(c) Prove that 0 ! = 1.

MIND think :

- $n\,! = n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times (n-5) \times \ldots\ 3 \times 2 \times 1$

9 Without using a calculator, find the value of each of the following. [Use a calculator, check the answer]

(a) ${}^{8}P_{8}$ =

(b) ${}^{8}P_{7}$ =

MIND think :

- ${}^{n}P_{r} = \dfrac{n\,!}{(n-r)\,!} \Rightarrow {}^{n}P_{r-1} = \dfrac{n\,!}{(n-r+1)\,!}$; ${}^{n}P_{r-2} = \dfrac{n\,!}{(n-r+2)\,!}$; ${}^{n-1}P_{r-1} = \dfrac{(n-1)\,!}{(n-r)\,!}$; . . .

  $\Rightarrow {}^{n}P_{r+1} = \dfrac{n\,!}{(n-r-1)\,!}$; ${}^{n}P_{r+2} = \dfrac{n\,!}{(n-r-2)\,!}$; ${}^{n-1}P_{r+1} = \dfrac{(n-1)\,!}{(n-r-2)\,!}$; . . .
- $n\,! = {}^{n}P_{n} = {}^{n}P_{n-1} \Rightarrow$ example : $8! = {}^{8}P_{8} = {}^{8}P_{7}$

(c) ${}^{10}P_{5}$ =

(d) ${}^{11}P_{4}$ =

10 Without using a calculator, find the value of each of the following. [Use a calculator, check the answer]

(a) ${}^{4}C_{0}$ =

(b) ${}^{10}C_{7}$ =

(c) ${}^{6}C_{2}$ =

(d) ${}^{6}C_{4}$ =

MIND think :

- ${}^nC_r = \dfrac{n!}{(n-r)!\,r!} = {}^nC_{n-r} \Rightarrow$ example : ${}^6C_2 = {}^6C_4$ ($2 + 4 =$)
- ${}^nP_1 = {}^nC_1 = {}^nC_{n-1} = n$
- $0! = 1! = {}^nP_0 = {}^nC_0 = {}^nC_n = 1 \Rightarrow {}^nP_r > 1 \sim r > 0$ *and* ${}^nC_r > 1 \sim 0 < r < n$

$\boxed{{}^nP_r = {}^nC_r\,[r!]}$ $\boxed{\dfrac{{}^nP_r}{{}^nP_{r-1}} = n - r + 1}$

⇒ ***solve problem involving*** $n!$, ${}^nP_r$, ${}^nC_r$

11 (a) Given $n! = {}^nP_r$, state the relation between n and r. [1 mark]

(b) Given ${}^nP_r \times k = {}^nP_{r+1}$, where $k > 0$. Express n in terms of r and k. (*Ans* : $n = k + r$)

[2 marks]

[*Forecast*]

Answer :

(a) (b)

12 Given ${}^{n+2}P_3 = 30n$, find the value of n. (*Ans* : 4)

[*Forecast*] [3 marks]

Answer :

13 Given ${}^{n+1}P_4 = 10\left({}^nP_2\right)$, find the value of n. (*Ans* : 4)

[*Forecast*] [3 marks]

Answer :

14 Given $\dfrac{{}^nP_3}{2m} = {}^{n-1}P_2$, express m in terms of n. (*Ans* : $m = \frac{n}{2}$)

[*Johor 2020*] [2 marks]

Answer :

15 Prove that : ${}^{n+1}P_r - {}^{n}P_r = r\,{}^{n}P_{r-1}$

[2 marks] [*Forecast*]

Answer :

16 If ${}^{2n}P_3 = 84n$, shows that ${}^{n+1}P_2 = \frac{20-n}{2}$.

[3 marks] [*Forecast*]

Answer :

17 (a) Given ${}^{6}C_n > 1$, list out all the possible values of n. [1 mark]

(b) Given ${}^{y}C_m = {}^{y}C_n$, express y in terms of m and n. [1 mark]

[*2018, No.2*]

Answer :

(a) (b)

18 Solve each of the following :

(a) Given ${}^{16}C_{r+3} = {}^{16}C_{7-r}$. (*Ans* : 2) [2 mark]

(b) Given ${}^{30}C_r = {}^{30}C_{r+2}$. (*Ans* : 14) [2 mark]

[*Forecast*]

Answer :

(a) (b)

19 (a) Without using table and calculator, find the value of

(i) ${}^{9}P_{3}$, (*Ans* : 504) [1 mark]

(ii) ${}^{9}C_{3}$. (*Ans* : 84) [1 mark]

(b) Hence, or using other methods, show that ${}^{9}P_{3} = {}^{9}C_{3} \times 3!$. [2 marks]

[*Forecast*]

Answer :

(a) (i) (b)

(ii)

20 Given ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ and ${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$.

(a) Show that ${}^{n}C_{r} = {}^{n}C_{n-r}$. [2 marks]

(b) Hence, without using table or calculator, find the value of ${}^{20}C_{3}$.
(*Ans* : 1140) [1 mark]

[*Forecast*]

Answer :

(a) (b)

21 Given ${}^{n}C_{6} = {}^{n}C_{8}$.

(a) Show that $n^2 - 13n - 14 = 0$ [2 marks]

(b) Hence, find the value of ${}^{n}C_{12}$. (*Ans* : 91) [2 marks]

[*Forecast*]

Answer :

(a)

(b)

22 Given $3\left({}^{n}C_{4}\right) = 5\left({}^{n-1}C_{5}\right)$, find the value of ${}^{n}C_{9}$. (*Ans* : 10)

[3 marks] [*Forecast*]

Answer :

⇒ ***determine the number of permutation of n different objects.***

⇒ ***determine the number of permutations of n different objects for given conditions.***

23 The diagram shows five cards of different letters.

H	E	B	A	T

(a) Find the number of possible arrangements, in a row, of all the cards. (*Ans* : 120)

(b) Find the number of these arrangements in which the letters E and A are side by side. (*Ans* : 48)

[4 marks] [***2004, No.23***]

Answer :

(a)

(b)

24 There are 4 different Science books and 3 different Mathematic books on a shelf. Calculate the number of different ways to arrange all the books in a row if

(a) no condition is imposed, (*Ans* : 5040)

(b) all the Mathematics books are next to each other. (*Ans* : 720)

[4 marks] [***2009, No.22***]

Answer :

(a)

(b)

25 In a group of 7 students, 4 are boys and 3 are girls. The students are requested to stand in a row. Find the number of ways to arrange them if

(a) a girl has to be seated at the centre, (*Ans* : 2160) [2 marks]

(b) the boys and girls are in alternate positions. (*Ans* : 144) [2 marks]
[*Forecast*]

Answer :

(a) (b)

MIND think :

- Permutations of n different objects $\Rightarrow$ *the number of ways* = $n!$ = ${}^{n}P_{n}$

26 The diagram shows five cards of different letters.

B	O	L	E	H

Calculate the number of different ways to arrange all the cards in a row if

(a) there is no restriction. (*Ans* : 120)

(b) the first card and the last card are consonants. (*Ans* : 36)
[3 marks] [*2016, No.3*]

Answer :

(a) (b)

27 The diagram shows six cards of different letters.

P	**U**	**T**	**E**	**R**	**A**

Find the number of arrangements of all the letters if

(a) no condition is imposed, (*Ans* : 720)

(b) the letters P and A are not together. (*Ans* : 480)
[*Forecast*] [4 marks]

Answer :

(a)

(b)

28 How many 5-digit numbers can be formed from the digits 0, 2, 5, 8, dan 9 without repeating any digit if each number end with digit 9 ? (*Ans* : 18)
[*Forecast*] [3 marks]

Answer :

⇒ ***Determine the number of permutations of n different objects taken r at a time.***

⇒ ***Determine the number of permutations of n different objects taken r at a time for given conditions.***

29 The diagram shows 5 letters and 3 digits.

A	B	C	D	E	6	7	8

A code is to be formed using those letters and digits. The code must consists of 3 letters followed by 2 digits. How many codes can be formed if no letter and digit is repeated in each code? (*Ans* : 360)
[3 marks] [***2003, No.22***]

Answer :

30 The diagram shows seven letter cards.

U	N	I	F	O	R	M

A four-letter code is to be formed using four of these cards. Find

(a) the number of different four letter codes that can be formed, (*Ans* : 840)

(b) the number of different four letter codes which end with a consonant. (*Ans* : 480)
[4 marks] [***2006, No.22***]

Answer :

(a) (b)

31 The diagram shows six numbered cards.

3	5	6	7	8	9

A four digit number is to be formed by using four of these cards. How many

(a) different numbers can be formed ? (*Ans* : 360)

(b) different odd numbers can be formed ? [***HINT*** ~ the last digit is odd] (*Ans* : 240)

[4 marks] [***2008, No.23***]

Answer :

(a) (b)

MIND think :

- Permutations of n different objects, taking r objects each time

  $\Rightarrow$ *number of ways* = ${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$; where $r \le n$

32 Diagram shows seven letter cards.

K	H	I	D	M	A	T

A five-letter code is to be formed using five of these cards. Find

(a) the number of different five-letter codes that can be formed, (*Ans* : 2520)

(b) the number of different five-letter codes which begin with vowel and end with a consonant.

(*Ans* : 600)

[4 marks] [*2011, No.23*]

Answer :

(a) (b)

33 How many 4-digit numbers can be form from the digits 0, 2, 4, 6 and 8 if no repetition is allowed ? (*Ans* : 96)

[2 marks] [*Forecast*]

Answer :

34 How many different numbers are greater than 5000 that can be formed from the numbers 3, 4, 5, 6 and 9 without repeatition. (*Ans* : 192)

[3 marks] [*Forecast*]

Answer :

35 Find the number of numbers between 1500 and 5000 can be formed from the digits 1, 2, 4, 5, 7, and 8 if each digit is used only once. (156)

[4 marks] [*Forecast*]

Answer :

36 Diagram shows five cards of different numbers.

1	**2**	**3**	**4**	**5**

(a) How many different numbers can be formed by using all or part of the cards ?
(*Ans* : 325) [2 marks]

(b) Based on (a), how many numbers formed are even ?

[***HINT*** ~ the last digit is even] (*Jwp* : 130) [4 marks]

[*Forecast*]

Answer :

(a)

(b)

37 The diagram shows six numbered cards.

1	5	6	7	8	9

A four digit number is to be formed by using four of these cards. How many

(a) different odd numbers that is greater than 7000 can be formed ? (*Ans* : 120)

(b) different odd numbers that is less than 6600 can be formed ? (*Ans* : 90)

[4 marks] [***Forecast***]

Answer :

(a)

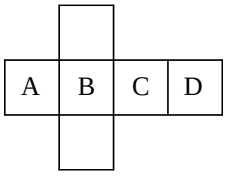
(b)

38 Diagram shows a four-figit passcode '0131' set by San on his smartphone.

He wants to reset the passcode such that the new passcode cannot consists of digit 1 followed by digit 3, and repetition of digits is not allowed. How many different passcosed can be formed? (*Ans* : 4872)
[3 marks] [*Forecast*]

Answer :

39 The diagram shows the net of a cube formed by Atana.



She wants to colour the faces of cube which are labelled up with letter A, B, C and D with the different colours using the colour of rainbow. Find the number of ways to colour the four faces if

(a) red colour must be used. (*Ans* : 480)

(b) red , green and blue colours must be used, (*Ans* : 96)
[4 marks] [*Forecast*]

Answer :

(a) (b)

40 A book rack can hold 6 books only.

(a) Find the number of arrangements to place six of 10 different books in the book rack.

(*Ans* : 151200)

(b) Find the number of these arrangements in which two certain books must be placed adjacent to each other. (*Ans* : 16800)
[4 marks] [*Forecast*]

Answer :

(a)

(b)

41 (a) Find the number of ways can 5 people to be seated in 6 seats in a row. (*Ans* : 720) [1 mark]

(b) There are 9 seats in a room, where 4 seats at the first row, and 5 seats at the second row. Find the number of ways to arrange 8 people in 9 seats if 3 particular persons must sit at the first row. (*Ans* : 17280) [3 marks]
[***Forecast***]

Answer :

(a)

(b)

42 The diagram shows two rows of chair. There are 4 seats at the first row, and 3 seats at the second row.

A group of people consisted of 2 children, 1 couple of husband-wife, and 4 adults. Find the number of ways of to arrange them in the 7 seats, if

(a) the couple of husband-wife and 2 children must sit in the same row, (*Ans* : 576)

(b) 2 children must sit in the same row, (*Ans* : 12960)
[5 marks] [***Forecast***]

Answer :

(a)

(b)

⇒ ***determine the number of arrangements of n objects taking r each time, when repetition is allowed.***

43 Diagram shows a four-figit passcode '0131' set by San on his smartphone.

He wants to reset the passcode such that the new passcode cannot consists of digit 1 followed by digit 3.
How many different passcosed can be formed ? (*Ans* : 9701)
[3 marks] [*2019, No.22*]

Answer :

44 Calculate the number of four digit numbers which have repeated digit. (*Ans* : 4464)
[*Forecast*] [3 marks]

Answer :

45 The diagram shows six numbered cards.

0	3	5	7	8	9

A four digit number is to be formed by using four of these cards. If the chosen card will be replaced with another card of the same number, how many

(a) different numbers can be formed ? (*Ans* : 1080)

(b) different odd numbers can be formed ? (*Ans* : 720)

(c) different odd numbers that is greater than 7000 can be formed ? (*Ans* : 432)

(d) different odd numbers that is less than 5500 can be formed ? (*Ans* : 192)
[7 marks] [***Forecast***]

Answer :

(a) (b)

(c) (d)

⇒ ***problems solving***

46 Diagram shows seven letter cards.

M	***I***	***N***	***E***	***R***	***A***	***L***

Five cards are chosen at random to form a code. Find the number of possible codes that can be formed which contains at least 3 consonants are arranged side by side. (*Ans* : 720)
[3 marks] [*clon2018, No.25*]

Answer :

47 A code is to be formed by using five letters from "***KEJITUAN***". Find the number of different codes that can be formed if the code

(a) begins with a vowel and ends with a consonant, (*Ans* : 1920) [2 marks]

(b) consists of 4 vowels, (*Ans* : 480) [2 marks]

(c) consists of 2 vowels and one of them located at the middle of the code. (*Ans* : 1152) [2 marks]
[***Forecast***]

Answer :

(a)

(c)

(b)

⇒ ***determine the number of permutations of n objects involving identical objects, and limited to one condition***

48 How many word codes can be formed using all the letters from the word '*CEMERLANG*' ?
(*Ans* : 181440)

[2 marks]

Answer :

49 Find the number of possible arrangements if the letters from the word '*STATISTIK*' is arranged in a row. (*Ans* : 15120)

[2 marks]

Answer :

50 Find the number of 7-digit odd numbers that can be formed from all the numbers 7, 2, 4 and 8, with all the numbers other than 7 appearing exactly twice. (*Ans* : 90)

[2 marks]

Answer :

51 The diagram shows five cards of different letters.

B	O	L	O	H

Calculate the number of different ways to arrange all the cards in a row if

(a) there is no restriction. (*Ans* : 60)

(b) the letters O and L are always together. (*Ans* : 18)

[3 marks] [*Forecast*]

Answer :

(a)

(b)

MIND think :

- Permutations for n objects involving identical objects

  $\Rightarrow$ *the number of ways* $= \dfrac{n!}{a!b!c!\ldots}$, where a, b, c, . . . are the number of identical objects for each type

52 The diagram shows five cards of different letters.

B	O	L	O	L

Calculate the number of different ways to arrange all the cards in a row if

(a) there is no restriction. (*Ans* : 30)

(b) the first card and the last card are consonants. (*Ans* : 9)

[3 marks] [*Forecast*]

Answer :

(a) (b)

53 The diagram shows seven cards of different letters.

S	U	A	S	A	N	A

Calculate the number of different ways to arrange all the cards in a row if

(a) there is no restriction. (*Ans* : 420)

(b) the vowels are always separated. (*Ans* : 12)

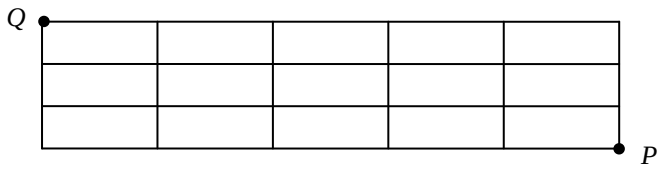
[3 marks] [*Forecast*]

Answer :

(a) (b)

⇒ ***solve problems involving counting the number of paths through a grid***

54 The diagram shows the routes for an object move from point P to point Q.

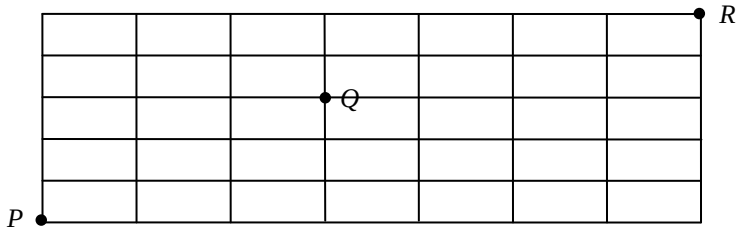


If the object can only move up or to the left, find the number of shortest routes for the object move from point P to point Q. (*Ans* : 56)

[2 marks] [*Forecast*]

Answer :

55 A man can only walk one step to the east or north.

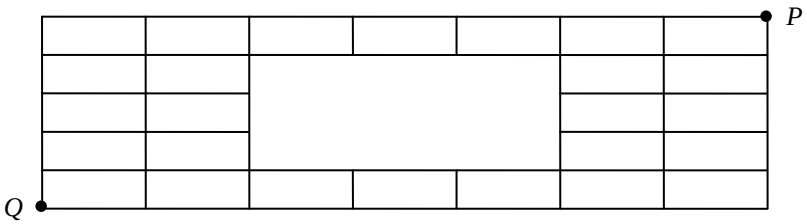


Find the number of different shortest route can be taken from P to R, via R. (*Ans* : 300)

[3 marks] [*Forecast*]

Answer :

56 An object can only move to the west or to the south.

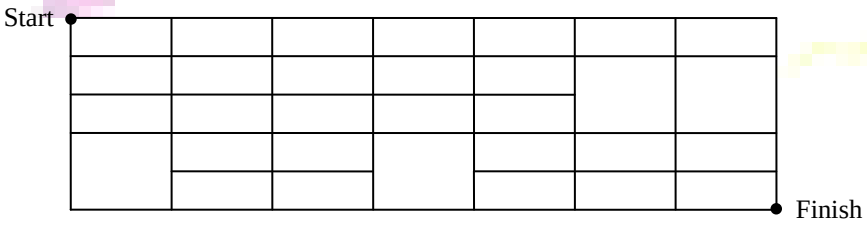


Find the number of paths for the object move from point P to point Q. (*Ans* : 192)

[3 marks] [*Forecast*]

Answer :

57 The diagram shows the paths for an object move from start point to finish point.



If the object can only move down or to the right, find the number of paths for the object move from start point to finish point. (*Ans* : 558)

[3 marks] [*Forecast*]

Answer :

⇒ ***determine the number of permutations of n objects taking r each time, involving identical objects and limited to one condition***

58 Diagram shows eleven letter cards.

M	**A**	**T**	**H**	**E**	**M**	**A**	**T**	**I**	**C**	**S**

A four-letter code is to be formed using these cards. Find the number of different four-letter codes that can be formed. (*Ans* : 2454)

[3 marks] [*Forecast*]

Answer :

59 The diagram shows sixteen numbered cards.

1	**1**	**2**	**2**	**2**	**3**	**3**	**4**	**4**	**4**	**5**	**6**	**7**	**7**	**8**	**9**

Find the number of 4-digit numbers that can be formed. (*Ans* : 4828)

[3 marks] [*Forecast*]

Answer :

60 Diagram shows seven letter cards.

K	H	I	A	M	A	T

A five-letter code is to be formed using five of these cards. Find

(a) the number of different five-letter codes that can be formed, (*Ans* : 1320)

(b) the number of different five-letter codes which begin with vowel and end with a consonant.
(*Ans* : 372)

[4 marks] [*Forecast*]

Answer :

(a)

(b)

61 Diagram shows seven letter cards.

M	I	N	E	R	A	R

Five cards are chosen at random to form a code. Find the number of possible codes that can be formed which contains at least 3 consonants are arranged side by side. (*Ans* : 360)

[3 marks] [*Forecast*]

Answer :

⇒ ***determine the number of permutation of n different objects in a circle.***
⇒ ***determine the number of permutations of n different objects, in a circle for for given conditions, limited to one condition.***

62 7 people are to be seated at a round table.

(a) Find the number of ways to arrange them. (*Ans* : 720)

(b) Find the number of ways to arrange them, if two certain people must sit next to each other. (*Ans* : 240)

[4 marks] **[*Forecast*]**

Answer :

(a) (b)

63 10 people are to be seated at a round table. Find the number of ways to arrange, if

(a) one of them is fixed. (*Ans* : 362880)

(b) three particular persons certain person among must sit together. (*Ans* : 30240)

[4 marks] **[*Forecast*]**

Answer :

(a) (b)

64 Find the number of ways can 6 persons be seated at a round table, if two particular peoples among them cannot sit next to each other. (*Ans* : 72)

[3 marks] **[*Forecast*]**

Answer :

65 5 boys and 5 girls are to be seated at a round table. Find the number of ways to arrange them, if the boys must always sit between the boys. (*Ans* : 2880)

[3 marks] **[*Forecast*]**

Answer :

MIND think :

- Circular permutations of n different objects $\Rightarrow$ *the number of ways* $= \dfrac{n!}{n} = \dfrac{n(n-1)!}{n} = (n-1)!$

66 There are 3 boys and 3 girls. Find the number of ways to arrange 6 of them to sit at a round table if at least 2 of the girls are sitting together. (*Ans* : 108)

[5 marks] **[*Forecast*]**

Answer :

67 There are 4 boys and 2 girls. Find the number of ways to arrange 6 of them to sit at a round table if :

(a) there is no condition impossed, (*Ans* : 120)

(b) the 2 girls must sit together, (*Ans* : 48)

(c) the 2 grisl must sit separately. (*Ans* : 72)

[5 marks] **[*Forecast*]**

Answer :

(a)

(b)

(c)

68 Four families, P, Q, R, S have 3, 3, 2, 2 members respectively. If all members of these families are seated around a circular table and members of same family must seated adjacently, find the number of ways of the seating arrangements. (*Ans* : 864)

[3 marks] **[*Forecast*]**

Answer :

⇒ ***determine the number of arrangements n different objects in a circular ring, chain, bracelet or necklace.***

69 Find the number of ways to assemble 11 beads of different colours to form a toy necklace.
(*Ans* : 1814400)

[2 marks] **[*Forecast*]**

Answer :

MIND think :

- Arrangements of n different objects to form a circular ring $\Rightarrow$ *the number of ways* $= \dfrac{(n-1)!}{2}$
  (do not involve clockwise or anticlockwise, because both are the same)

70 There are 8 distinct coloured beads.

(a) Find the number of ways to arranged the beads at a round table. (*Ans* : 5040)

(b) Find the number the beads be strung on a circular ring. (*Ans* : 2520)

[3 marks] **[*Forecast*]**

Answer :

(a) (b)

⇒ ***determine the number of permutations of n different objects talking r objects each time, and arranged in a circle.***

71 A group of 9 children are competing for 8 chairs that are arrange in a cirlce during a musical chair game. The children have to move in a clockwise direction aroud the chair. Find the number of arrangements for this game. (*Ans* : 45360)

[2 marks] **[*Forecast*]**

Answer :

72 Nicole bought 10 different young trees to decorate the mini garden at her house. Due to the limited space, she can only plant 6 trees in a circle. Find the number of ways in which Nicole can plant the young trees. (*Ans* : 25200)

[2 marks] **[*Forecast*]**

Answer :

73 There are 4 boys and 3 girls. Find the number of ways to arrange 6 of them to sit at a round table if :

(a) there is no condition impossed, (*Ans* : 840)

(b) all girls must be seated, (*Ans* : 480)

(c) only 2 girls are seated. (*Ans* : 360)

[5 marks] **[*Forecast*]**

Answer :

(a)

(b) (c)

MIND think

- Circular permutations of n different objects taking r each time $\Rightarrow$ *the number of ways* $= \dfrac{{}^{n}P_{r}}{r}$, where $r \le n$

$\Rightarrow$ ***determine the number of permutations of n different objects talking r objects each time, in a circular ring, bracelet or necklace.***

74 Juliet bought 11 beads of different colours. She intends to make a bracelet. Juliet realises that she only needs 8 beads. Find the number of ways to make the bracelet. (*Ans* : 415800)

[2 marks] **[*Forecast*]**

Answer :

75 There are 10 beads with different colours. How many number of ways are there to make a bracelet if :

(a) there is no condition impossed, (*Ans* : 181440)

(b) only 8 beads will be used. (*Ans* : 113400)

[3 marks] **[*Forecast*]**

Answer :

(a) (b)

MIND think :

- Arrangements of n different objects taking r each time, to form a circular ring

  $\Rightarrow$ *the number of ways* $= \dfrac{{}^{n}P_{r}}{2r}$, where $r \le n$

⇒ ***determine the number of permutations of n objects involving identical objects, in a circle.***

76 Letters from the word " **DAYANG** " are arranged aroud a circle. Find the number of possible arrangements. (*Ans* : 60)

[3 marks] **[*Forecast*]**

Answer :

77 There are 2 red beads, 2 blue beads and 1 green bean. Find the number of ways to arrange them in a circle. (*Ans* : 6)

[3 marks] **[*Forecast*]**

Answer :

78 13 stalks of flowers consisting of 3 red flower, 4 blue flowers, and the remain are white flowers. The flowers will be arranged at a round table. Calculate the number of ways to arrange the flowers. (*Ans* : 4620)

[3 marks] **[*Forecast*]**

Answer :

MIND think :

- Arrangements of n different objects involving identical objects, in a circle

  $\Rightarrow$ *the number of ways* $= \dfrac{(n-1)!}{a!b!c!\ldots}$, where a, b, c, . . . are the number of identical objects for each type

4.2 *Combination*

4.2.1 Compare and contrast permutation and combination.

4.2.2 Determine the number of combinations of r objects chosen from n different objects at a time.

4.2.3 Solve problems involving combinations with certain conditions

---

79 (a) State the value of ${}^{n}C_{0}$. **

(b) Three students are to be selected from 5 boys and 4 girls to participate in a competition. Find the number of different ways to choose the participants if :

(i) all the three students are boys, (*Ans* : 10)

(ii) one boy and two girls are chosen. (*Ans* : 30)

[3 marks] [***2015, No.21***]

Answer :

(a)

(b) (i) (ii)

80 A debating team consists of 5 students. These 5 students are chosen from 4 monitors, 2 assistant monitors and 6 perfects. Calculate the number of different ways the team can be formed if :

(a) there is no restriction, (*Ans* : 792)

(b) the team consists only 1 monitor and exactly 3 perfects. (*Ans* : 160)

[4 marks] [***2005, No.22***]

Answer :

(a) (b)

81 (a) State the value of ${}^{n}P_{1} - {}^{n}C_{1}$. ** [1 mark] [*Forecast*]

(b) There are 6 sweets, each with a different flavour, which are to be divided equally between 2 children. Find the number of different ways the division of the sweets can be done.
(*Ans* : 20)
[3 marks] [*2010, No.23*]

Answer :

(a) (b)

MIND think :

• Combinations of r objects chosen from n different objects, (without considering the positions or arrangements)

$\Rightarrow$ *the number of ways* $= {}^{n}C_{r} = \dfrac{n!}{(n-r)!\,r!} = \dfrac{{}^{n}P_{r}}{r!}$, where $r \le n$.

82 A badminton team consists of 7 students. The team will be chosen from a group of 8 boys and 5 girls. Find the number of teams that can be formed such that each team consists of

(a) 4 boys, (*Ans* : 700)

(b) not more than 2 girls. (*Ans* : 708)
[4 marks] [***2003, No.23***]

Answer :

(a) (b)

83 (a) State the value of ${}^{n}C_{n}$. ** [1 mark] [*Forecast*]

(b) There are 10 different coloured marbles in the box. Find

(i) the number of ways 3 marbles can be chosen from the box, (*Ans* : 120)

(ii) the number of ways at least 8 marbles can be chosen from the box. (*Ans* : 56)
[4 marks] [*2012, No.23*]

Answer :

(a)

(b) (i) (ii)

84 A group of 5 students are to be chosen from 9 boys and 7 girls to form a school debate team. Find the number of different ways to form the team which consist of

(a) girls only, (*Ans* : 21)

(b) at least 4 boys. (*Ans* : 1008)

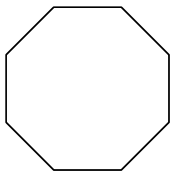
[4 marks] [***2014, No.23***]

Answer :

(a)

(b)

85 The diagram shows a regular octagon.



Using the vertices of the polygon, find the number of

(a) straight lines that can be drawn, [*HINT* ~ choose 2 points] (*Ans* : 28)

(b) triangles that can be formed, [*HINT* ~ choose 3 points] (*Ans* : 56)

(c) quadrilaterals that can be formed. [*HINT* ~ choose 4 points] (*Ans* : 70)

[4 marks] [***Forecast***]

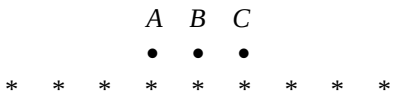
Answer :

(a)

(b)

(c)

86 Diagram shows 12 points plotted on two parallel rows.



First row has three points, which are A, B, and C. Second row has 9 points. Triangle is to be formed by joining the points. Find the possible number of triangles that can be formed if

(a) no condition is impossed, (*Ans* : 135) [2 marks]

(b) (i) point B and point C are the vertices of the triangle. (*Ans* : 9) [2 marks]

(ii) point A is one of the vertices of the triangle. (*Ans* : 54) [2 marks]
[***Forecast***]

Answer :

(a)

(b) (i)

(ii)

87 By using combination, find the number of diagonals that can be drawn in a convex 9-sided polygon. (*Ans* : 27)
[3 marks] [***Forecast***]

Answer :

88 There are 9 points on a plane. Except for 4 collinear points, no others points lie in a straight line. How many straight lines can be formed from these 9 points ? (*Ans* : 31)
[3 marks] [***Forecast***]

Answer :

89 In a ping-pong match, how many different sets of mixed-doubles can be arranged form 4 men and 4 women ? (*Ans* : 72)
[3 marks] [***Forecast***]

Answer :

90 There are 6 boys and 8 girls badminton players. How many different mixed-double matches can be held ? [*HINT* ~ choose 2 boys and 2 girls] (*Ans* : 840)

[3 marks] [***Forecast***]

Answer :

91 Jeovy has 6 flowers, each of a different variety. How many different bouquets that can he form ? (*Ans* : 63)

[3 marks] [***Forecast***]

Answer :

92 A Mathematics test consists of two sections. The number of questions in section *A* is 5, whereas the number of questions in section *B* is 4. Find the number of ways a student can answer the questions in the test if the student must answer :

(a) any 6 questions, (*Ans* : 84)

(b) 4 questions from section *A* and 2 questions from section *B*. (*Ans* : 30)

[4 marks] [***Forecast***]

Answer :

(a) (b)

93 A team of 20 rescue volunteers for earthquake victims is to be set up. There are 8 doctors, 10 nurses and 12 workers volunteering their services. Find the number of ways the team can be formed if

(a) 3 doctors, 6 nurses and 11 workers are required, (*Ans* : 141120)

(b) at least 2 doctors are required. (*Ans* : 30032464)

[4 marks] [***Forecast***]

Answer :

(a) (b)

94 A badminton team consists of 5 boys and 5 girls. The team will be chosen from a group of 8 boys and 9 girls. Find the number of teams that can be formed such that

(a) a certain boy and girl must be chosen, (*Ans* : 2450)

(b) two certain girls are either both of them to be chosen or not. (*Ans* : 3136)

[4 marks] [***Forecast***]

Answer :

(a) (b)

95 A committee consists of 5 members to be selected from 8 men and 7 women. Find the number of possible committees that can be formed if the men is the majority. (*Ans* : 1722)

[3 marks] [*Forecast*]

Answer :

96 A parcel consists of 4 items which is chosen from 7 books and 5 pens. Find the number of different parcels that can be prepared such that each parcel consists of

(a) 1 book only, (*Ans* : 70)

(b) books and pens, and the pens are not more than two. (*Ans* : 385)

[4 marks] [***Forecast***]

Answer :

(a) (b)

97 A Kenari car can accommodate 1 driver and 3 adults. Find the number of different ways the selections can be made from 3 men and 4 women if

(a) there is no restriction for the seating, (*Ans* : 35)

(b) the driver must be a man. (*Ans* : 60)

[4 marks] [***Forecast***]

Answer :

(a) (b)

98 SM St. Peter Telipok is organising a handball competition. There are 12 teams participate in the competition. In the first round, 12 teams will be divided equally into 3 groups. Every teams in the group will compete with other teams within the same group. Find

(a) the number of ways to divide the 12 teams into 3 groups, (*Ans* : 34650) [2 marks]

(b) the number of matches that need to be done in the first round. (*Ans* : 18) [2 marks]

[***Forecast***]

Answer :

(a) (b)

99 A committee which consists of 3 members need to be formed from four married couples. Find the number of ways to form the committee if

(a) only one woman in the committee, (*Ans* : 24)

(b) husband and wife cannot be chosen together as the members of the committee. (*Ans* : 32)

[4 marks] [***Forecast***]

Answer :

(a) (b)

⇒ ***Permutation + Combination***

100 A coach wants to choose 5 players consisting of 2 boys and 3 girls to form a badminton team. These 5 players are chosen from a group of 4 boys and 5 girls. Find

(a) the number of ways the team can be formed, (*Ans* : 60)

(b) the number of ways the team members can be arranged in a row for a group photograph, if the three girls sit next to each other. (*Ans* : 36)

[4 marks] [***2007, No.23***]

Answer :

(a) (b)

101 The diagram shows a seven-letter word.

C	**L**	**U**	**S**	**T**	**E**	**R**

(a) Find the number of different ways to arrange all the letters in a row. (*Ans* : 5040)

(b) Four letters are to be chosen from the word. Find the number of ways of choosing the four letters which consists of 3 consonants. (*Ans* : 20)

[3 marks] [***2013, No.23***]

Answer :

(a) (b)

102 Dahlia has a home decoration shop. One day, Dahlia received 14 sets of cups from a supplier. Each set contained 6 pieces of cups of different colours.

(a) Dahlia chooses 3 sets of cups at random to be checked. Find the number of different ways that Dahlia uses to choose those sets of cups. (*Ans* : 364)

(b) Dahlia takes a set of cups to display by arranging it in a row. Find the number of different ways the cups can be arranged such that the blue cup is not placed next to the red cup. (*Ans* : 480)

[4 marks] [*2017, No.22*]

Answer :

(a)

(b)

103 The table shows the number of girls needed to manage three stalls in a school carnival day.

Types of stall	Food	Drink	Game
Number of student needed	4	2	3

On that day, 9 students were assigned to manage the stalls.

(a) How many different ways the students can be assigned. (*Ans* : 1260)

(b) The school wants to reward the students. A student is chosen at random as a leader among them to lead all the recipients. The chosen leader and her group member will lead the reception followed by the other two groups. The students from the same groups have to line up together. How many different ways can the students be arranged ? (*Answer* : 1728)

[4 marks] [*2020, No.23*]

Answer :

(a)

(b)

104 A teacher wants to choose 7 students consisting of 3 boys and 4 girls to form a committee of Science and Mathematics Club. These 7 students are chosen from 5 boys and 6 girls. Find the number of ways

(a) the committee can be formed, (*Ans* : 150)

(b) the committee can be arranged at a round table in the meeting if the four girls sit next to each other. (*Ans* : 144)
[4 marks] [***Forecast***]

Answer :

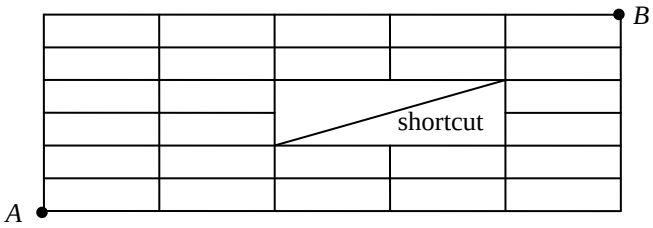
(a) (b)

CONTINUOUS EXERCISES

105 Six people A, B, C, D, E, F are to be seated in a row. If F be seated on the right-hand side of E, but E and F cannot sit next to each other, find the number of different way of the seating. (*Ans* : 240)
[3 marks]
[*Forecast*]

Answer :

106 An object can only move up or to the right.



Find the number of shortest routes for the object move from A to B, through the shortcut. (*Ans* : 18)
[3 marks] [*Forecast*]

Answer :

107 10 persons are invited to a party. Find the number of way they can be seated in a round table such that two particular persons sit on either side of the host. (*Ans* : 80640)
[3 marks] **[*Forecast*]**

Answer :

108 If the number of linear permutations for n distinct objects is six times the circular permutations of the objects. Find the number of circular permutations for the objects. (*Ans* : 120)
[3 marks] **[*Forecast*]**

Answer :

109 Find the number of 8-digit numbers that can be formed from all the numbers 0, 2, 3, 3, 4, 4, 5, and 6, without repeating any digit. (*Ans* : 8820)
[3 marks] **[*Forecast*]**

Answer :

110 The diagram shows seven numbered cards.

0	2	3	4	5	8	9

Five digit numbers are to be formed by using five of these cards without repetition.

(a) How many even numbers can be formed ? (*Ans* : 1260)

(b) How many of the number formed in (a) are greater than 40000 ? (*Ans* : 840)
[4 marks] [***Forecast***]

Answer :

(a) (b)

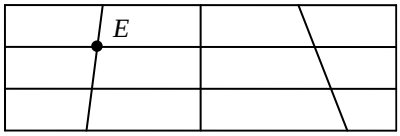
111 How many 4-digit numbers, which are divisible by 4 can be arranged from the numerals 0, 1, 2, 3, 4, and 5, without repeating any digit. [*HINT* ~ the last 2 digits is divisible by 4] (*Ans* : 72)
[3 marks] **[*Forecast*]**

Answer :

112 How many 5-digit numbers, which are divisible by 25 can be arranged from 0, 1, 2, 3, 4, 5, 6 and 7, without repeating any digit. [*HINT* ~ the last 2 digits is 00 or divisible by 25] (*Ans* : 320)
[3 marks] **[*Forecast*]**

Answer :

113 The diagram shows a grid.



Find the number of quadrilaterals found in diagram if

(a) no condition is imposed, [***HINT*** ~ choose 2 vertical and 2 horizontal] (*Ans* : 60)

(b) the point E is on the vertices of quadrilaterals. (*Ans* : 12)

[3 marks] **[*Forecast*]**

Answer :

(a)

(b)

PROBABILITY DISTRIBUTION

- ONE PAGE NOTE (OPN)
- WORKSHEET

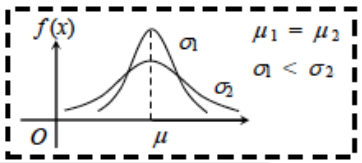
Puan Jamaliah Mohd Elmi

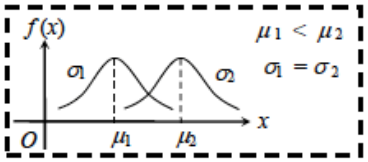
NOTA ONE PAGE (dengan pengekodan)

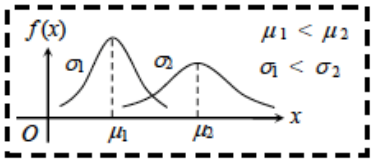
" PROBABILITY DISTRIBUTION "

$P(|z|<a) = P(-a<z<a) = 1-2P(z>a)$

$P(|z|>a) = P(z<-a)+P(z>a) = 2P(z>a)$







BINOMIAL DISTRIBUTION

$X \sim B(n, p)$ → X discrete random variable (DRV) ; $X = 0, 1, 2, 3, 4, 5, \ldots, n$

$P(X=r) = {}^nC_r\ p^r\ q^{n-r}$ ~ r = number of success ($r = 0, 1, 2, 3, \ldots\ldots, n$)

~ n = number of trial ①

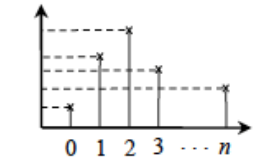
~ p = probability of success ($0<p<1$)

~ q = probability of failure ($q = 1-p$ @ $p+q=1$)

percentage

÷ 100 ↓ ↑ × 100

probability



$^nC_0 = 1,\ ^nC_n = 1\ ;\ ^nC_1 = n\ ;\ a^0 = 1$

➲ $^nC_r = \frac{n!}{(n-r)!\,r!} \Rightarrow {}^nC_r = {}^nC_{n-r}$

➲ $n! = n\times(n-1)\times(n-2)\times\ldots\times3\times2\times1$

$P(X=0) + P(X=1) + P(X=2) + P(X=3) + \ldots\ldots\ldots + P(X=n) = 1$

② ↳ $P(X<2) = P(X=0) + P(X=1)$

③ ↳ $P(X\geq2) = 1 - P(X=0) - P(X=1)$

- mean, $\mu = np$
- variance, $\sigma^2 = npq$ ④
- standard deviation, $\sigma = \sqrt{npq}$

Notes :

➲ less than (<)

➲ at most, not more than, (≤)

➲ more than (>)

➲ at least, not less than (≥)

Find $n(A)$ & $n(S)$ ②

[Find the probability first]

$P(A) = \frac{n(A)}{n(S)}$

calculator *fx*-570MS

mode ; mode ; 1 ;

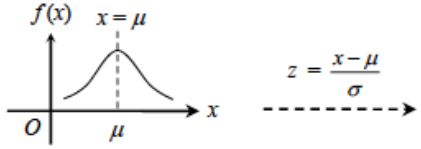
shift ; 3

R ~ >, ≥ & P ~ <, ≤

NORMAL DISTRIBUTION

$X \sim N(\mu, \sigma^2)$ → X continuous random variable (CRV) ; $X<a$, $X>a$, $a<$

Normal distribution, $X \sim N(\mu,$ Standardized normal distribution, $Z \sim N(0,$

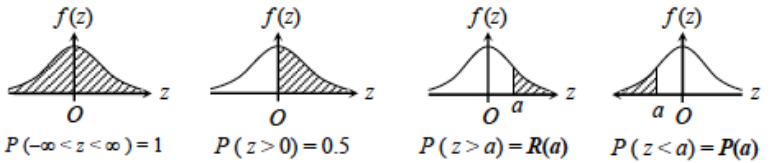


$\mu = 0$

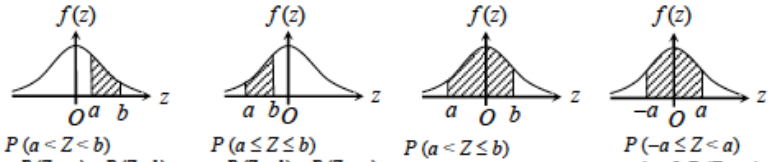
$\sigma^2 = 1$

$\sigma = 1$

Find probability [given x-score, convert to z-score, find probability] ①



$P(-\infty<z<\infty) = 1$ $P(z>0) = 0.5$ $P(z>a) = R(a)$ $P(z<a) = P(a)$



$P(a<Z<b) = P(Z>a) - P(Z>b)$

$P(a\leq Z\leq b) = P(Z<b) - P(Z<a)$

$P(a<Z\leq b) = 1 - P(Z<a) - P(Z>b)$

$P(-a\leq Z<a) = 1 - 2P(Z>a)$

Find X-score [get the probability, find z-score, convert to X] ③

⟪ **EXAMPLE** ⟫

- $P(x>k) = 0.4$

$P(z>\frac{k-\mu}{\sigma}) = 0.4$

~ $P(z>\underline{0.253}) = 0.4$ [from table]

$\frac{k-\mu}{\sigma} = 0.253$

- $P(x>k) = 0.87$

$P(z<\frac{k-\mu}{\sigma}) = 0.13$

~ $P(z>\underline{1.127}) = 0.13$ [from table]

$\frac{k-\mu}{\sigma} = -1.127$

WORKSHEET
TOPIC : PROBABILITY DISTRIBUTION

BINOMIAL DISTRIBUTION FORMULAE

1. $P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$
2. Mean / *Min*, $\mu = np$
3. $\sigma = \sqrt{npq}$

NORMAL DISTRIBUTION FORMULAE

$$Z = \frac{X - \mu}{\sigma}$$

A. BINOMIAL DISTRIBUTION

1. The probability that it will rain in a particular day is $\frac{3}{5}$. Find the probability that in a particular week, it will rain for

(a) exactly 4 days, [0.2903]
(b) more than 5 days. [0.1586]

[(a) 0.2903; (b) 0.1586]

2. The probability of getting a bad orange from a box is 0.2. Christina has chosen 6 oranges. Calculate the probability that

(a) only one orange is bad.
(b) all the oranges are good.

[(a) 0.3932; (b) 0.2621]

3. In one report, 54% of Malaysians buy locally made cars. If 8 peoples who just bought new cars are selected at random, find the probability that

(a) at least 2 of them bought locally made cars,
(b) more than 6 of them bought locally made cars.

[(a) 0.9792; (b) 0.05650]

4. In a housing estate, 7 out of 10 families own a car. If a sample of 8 families is selected, find the probability that the number of family owning a car is
(a) exactly 5 families,
(b) at least 2 families.

[(a) 0.2541; (b) 0.9987]

5. In a survey, it is found that 25% of the bulbs in a particular shop are spoilt. Find the probability that out of 10 bulbs,
(a) exactly 3 bulbs are spoilt,
(b) more than 8 bulbs are in **good** condition.

[(a) 0.2503; (b) 0.2440]

6. Probability that Bob will win in a chess competition is $\frac{3}{4}$. If he plays five games, calculate the probability that he will win
(a) exactly four games.
(b) more than three games..

[(a) 0.3955 ; (b) 0.6328]

7. The probability that a school bus will be late to school is 0.15. Find the probability that in 5 school's days, the bus will be late
(a) exactly 2 days,
(b) at least once.

[(a) 0.1382 ; (b) 0.5563]

8. The probability that a shooter hits the target is 0.8. If he fires 9 shots, find the probability that he will hit the target
(a) exactly 8 times.
(b) at least 8 times.

[(a) 0.3020; (b) 0.4362]

9. In a certain company, there are 3 out of 20 employees randomly selected, have life insurance. If there are 600 employees in the company, estimate the number of employees whose have the life insurance. Hence, calculate the variance and the standard deviation.

[90; 76.5; 8.746]

10. A discrete random variable X has a binomial distribution, which is $X \sim B(n, p)$ with a mean of 45 and a standard deviation of 3. Find the values of n and p.

$[n = 56; p = 0.8]$

11. A discrete random variable $X \sim B(120, 0.4)$. Find its mean and standard deviation.

[48; 5.367]

12. If X is a Binomial random variable $X \sim B(5, 0.45)$, find
(a) The mean, variance and standard deviation of X.

(b) $P(X=4)$ and $P(X>3)$.

[(a) 2.25, 1.2375, 1.1124 ; (b) 0.1128, 0.1313]

13. Given that X is a binomial random variable with a mean and standard deviation of 4 and $\sqrt{3}$ respectively, calculate $P(X=0)$.

[0.01002]

14. Given $X \sim B(n, 0.2)$. If $P(X=3)=P(X=4)$, find the numerical value of n, and hence, calculate $P(X=0)$.

[Hint : Use ${}^nC_3 = \dfrac{n(n-1)(n-2)}{3\times 2\times 1}$ and ${}^nC_4 = \dfrac{n(n-1)(n-2)(n-3)}{4\times 3\times 2\times 1}$]

[$n = 19, 0.01441$]

15. There are 5 000 people in a village. It is found that 8 out of 10 of the villagers installed broadband at home. Find the mean, variance and standard deviation for the number of people who have broadband at home.

[4 000; 800; 28.28]

16. In a study, it is found that 3 out of 5 men enjoy watching football games. If 1 000 men are randomly selected, find the mean and the standard deviation for the number of men who enjoy watching football games.

[600; 15.49]

17. In a game, participants have to guess the number of marbles in a bottle. The probability of guessing correctly is p.

(a) Find the value of p and the number of guesses so that the mean and the variance are 36 and 144 respectively.

(b) If a participant can make eight guesses, find the probability that four of them are correct.

[(a) 0.6 , 60 ; (b) 0.2322]

18. 80% of pupils in a certain school are interested in science. A sample consists of *n* pupils are randomly selected from the school.

(a) If the probability that all the pupils selected are interested in science is 0.1342, find the value of n.
(b) Based on the answer in (a), find the probability that there are less than three pupils interested in science.

[(a) 9 ; (b) 3.139×10^{-4}]

19. 7 students at a local university applied for state foundation scholarships. The probability that a student is awarded the scholarship is Find the probability that
(a) all of them are awarded the scholarships,
(b) only two students are awarded the scholarships,
(c) at most two students are awarded the scholarships.

[(a) $\frac{1}{2187}$; (b) 0.3073; (c) 0.5707]

NORMAL DISTRIBUTION

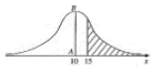
INOVATION OF TAB 4Y

HOW TO USE THE 'TAB 4Y' TECHNIC TO SOLVE PROBLEMS INVOLVING THE PROBABILITY OF NORMAL DISTRIBUTION.

METHOD 1: Given *X*, find the probability, *P*.. → Solve from left to right – Use Calculator

STEP 1	STEP 2	STEP 3	STEP 4
X	***Z***	< Graf >	***P(X ≤ x) atau P(X ≥ x)***
Write the value of *X* (Normal reading)	Convert *X* to *Z* by using the formula $Z = \frac{X - \mu}{\sigma}$	 Sketch the Standardized Normal graph	Use calculator to find the probability, write the answer.

METHOD 2: Given the probability, *P*, find the X. ← Solve from right to left – Find z-score from Table

STEP 4	STEP 3	STEP 2	STEP 1
X	***Z***	< Graf >	***P(X ≤ x) atau P(X ≥ x)***
Convert the z-score to X by using the formula $Z = \frac{X - \mu}{\sigma}$	Find the z-score from Standardized Normal Distribution Table	 Sketch the Standardized Normal graph to determine the area	Write the value of probability

Example:

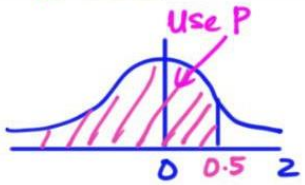
The marks obtained by a group of students are normally distributed with a mean of 45 marks and a variance of 36 marks.

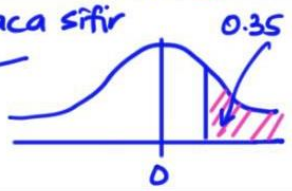
(a) If a student is chosen at random, calculate the probability that his mark is less than 8.

(b) Given that 35% of the students obtain more than *k* marks, find the value of *k*.

Solution :

Penyelesaian : $\mu = 45,\ \sigma = 6$ (use calculator)

(a)	X →	Z →	Graf →	P(X < 48)
	48	$Z = \frac{48-45}{6} = 0.5$		= 0.6915

(b)	X ←	Z ←	Graf ←	P(x > k)
	$0.385 = \frac{k-45}{6}$, $k = 47.31$	0.385 ←	Baca sifir 	0.35

[(a) 0.6915 ; (b) $k = 47.31$]

20. The marks obtained by the students in a test are normally distributed with a mean of 72 and a standard deviation of 15. A student is selected randomly, find the probability that the marks obtained by the student are

(a) less than 68.

(b) between 69 and 78.

[(a) 0.3947 ; (b) 0.2347]

21. The ages of 80 000 residents in a town are normally distributed with a mean of 45 years and variance of 36 years^2.

(a) Calculate the probability that the age of resident, selected randomly, is more than 48 years.
(b) Estimate the number of residents who are above 40 years.

[(a) 0.3085 ; (b) 63806]

22. The lifespans of a type of batteries are normally distributed with a mean of 150 hours and a variance of 25 hours.

(a) Find the probability that a battery selected at random has a lifespan between 140 hours and 148 hours.
(b) Given 10% of the batteries have a lifespan of more than x hours. Find the value of x.

[(a) 0.3218 ; (b) 156.4]

23. The mass of a group of students is normally distributed with a mean of 56 kg and a standard deviation of 8 kg.
(a) Find the mass of a student if the standard score of his mass is 1.75.

(b) If a student is chosen at random, find the probability that the student has a mass of more than 63 kg.

[(a) $X = 70$; (b) 0.1908]

24. The lifespan of bulbs produced by a factory in a day is normally distributed with a mean 60 hours and a standard deviation of 12 hours.

(a) Find the percentage of bulbs which have lifespan of more than 75 hours.
(b) If 85 bulbs have the lifespan of more than 75 hours, find the total number of bulbs produced by the factory in that day (to the nearest integer).

[(a) 10.56% ; (b) 805]

25. The scores of 14 500 candidates in an examination are normally distributed with a mean of 58 and a standard deviation of 16.
(a) If the minimum score to get distinctions is 70, estimate the number of candidates who score distinction in the examination.
(b) If 5 800 candidates have scores more than h, find the value of h.

[(a) 3286 ; (b) $h = 62.05$]

26. Statistics of health inspection from a school which has 82 teachers shows that the blood pressure of the teachers distributed normally with a mean of 130 mmHg and standard deviation of 18 mmHg. If the blood pressure more than 150 mmHg is considered as high blood pressure, find the number of teachers whose have high blood

pressure.

[11]

27. The thickness of the electric component produced by a machine has normally distributed with a mean of 0.5 mm. The thickness of the electric component which less than 0.47 mm are considered as unsatisfied the quality control and will be rejected. If 3.5% of the electric components are rejected, find the standard deviation for the distribution.

[0.0166]

28. The diameter of a screw produced by a factory are normally distributed with a mean of 2 cm and variance of 0.0025 cm. If 2.5% of the screws have the diameter less than p cm, and 4% have the diameter less than q cm, find the value of p and of q.

$[p = 2.098; q = 1.912]$

29. The masses of eggs produced by a farm follow a normal distribution. If 49.2% of the eggs have a mass more than 60 g and 23.8% have a mass less than 40 g, find the mean and the standard deviation of the eggs.

[(a) 59.45 g ; (b) 27.50 g]

30. The marks obtained by a group of 600 students in the Maths' Quiz are normally distributed with a mean of 68 and standard deviation of 11.

(a) If 30% of the students failed in the quiz, find the minimum marks needed to be pass.

(b) If 10 students got excellent marks in the quiz, find the minimum marks needed to be excellent in the quiz.

[(a) 63; (b) 92]

31. X is a normal random variable such that $X \sim N(\mu, \sigma^2)$. Find, correct to 4 decimal places,

(a) $P(|X - \mu| < 0.75\sigma)$,

(b) the value of μ and σ given that $P(X > 8.4) = 0.70$ and $P(X > 13.2) = 0.25$.

[(a) 0.5468; (b) 10.50, 4.007]

32. A factory produces a type of needles which the length normally distributed with a mean of 4 cm and the standard deviation of 0.05 cm.

(a) If a needle is randomly selected, find the probability that the length is less than 3.9 cm.

(b) If the factory produced 2000 pieces of needles a day, find the number of needles which the length is more than 4.1 cm.
(c) Find the percentage of the needles which the length is between 3.95 cm and 5.05 cm.

[(a) 0.0228; (b) 46; (c) 0.6826]

33. A poultry farm produces 10 000 chicken eggs daily. The girths or size of the eggs are of a normal distribution with mean girth 7.60 cm and standard deviation 0.50 cm. 10% of the largest eggs, which are considered grade A, are exported, whereas 5% of the smallest eggs, which are considered as grade C, are sold to bakeries for use in cakes and pastries. The rest of the eggs, which are considered grade B, are sent to supermarkets for domestic consumption.
(a) Find the range of the girths of grade B eggs.
(b) Find the daily production of grade B eggs.
(c) If 10 eggs from the farm are chosen at random, find the probability that exactly 6 are grade B eggs.

[(a) 6.7775 cm to 8.241 cm; (b) 8500 eggs; (c) 0.0401]

34. An actuary researched the effect of increased mean life expectancy or insurance policy holders on the profitability of an insurance company. The actuary makes a calculated assumption that the life expectancy of policy holders has a normal distribution, with mean life expectancy of 60 years and standard deviation of 14 years. The company would lose money on any policy holder who passes away before the age of 50 years.

(a) Show that the probability that the insurance company would make a profit from any one of its policy holders chosen at random is 76.24%.
(b) If 6 policy holders are chosen at random, find the probability that the insurance company makes a profit from at least 5 of them.
(c) Determine the minimum number of policy holders that needs to be randomly chosen so that the probability of the insurance company making a profit from at least one of them is greater than 98.90%.

[(b) 0.5633; (c) 4]

SPM QUESTIONS

PAPER 1

35. SPM 2010 P1 Q25

The discrete random variable X has a binomial probability distribution with $n = 4$, where n is the number of trials. Diagram 25 shows the probability distribution of X.

Find

(a) the value of k,

(b) $P(X \geq 3)$ [4 marks]

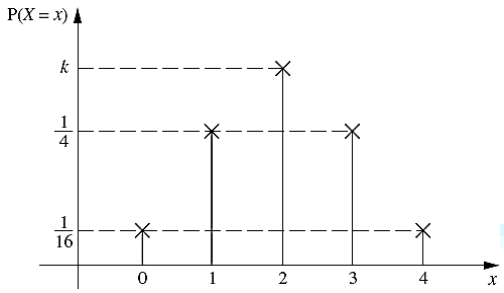


Diagram 25

[(a) $\frac{3}{8}$; (b) $\frac{5}{16}$]

36. SPM 2011 P1 Q25

Diagram 25 shows graph of binomial distribution X.

Find

(a) $P(X \geq 1)$

(b) The value of m. [3 marks]

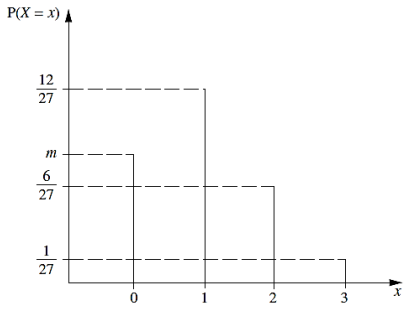


Diagram 25

[(a) $[\frac{19}{27}]$; (b) $[\frac{8}{27}]$]

37. SPM 2012 P1 Q25

In a test, 60% of the students has passed. A sample of 8 students is chosen at random. Find the probability that more than 6 students from the sample passed the test.

[3 marks]

[0.1064]

38. SPM 2013 P1 Q25

The random variable X represents a binomial distribution with 10 trials and the probability of success is $\frac{1}{3}$. Find

(a) the standard deviation of the distribution.
(b) the probability that at least one trial is success.

[4 marks]

[(a) 1.4907 ; (b) 0.9827]

39. SPM 2016 P1 Q2

Two fair coins are tossed simultaneously. H denotes the event of obtaining the head and T denotes the event of obtaining the tail.

(a) List the sample space using set notation.
(b) Given X is a discrete random variable which represents the number of heads obtained, list the possible values of X.

[2 marks]

[(a) $S = \{HH, HT, TH, TT\}$; (b) $X = 0, 1, 2$]

40. SPM 2016 P1 Q24

A sample of 3 light bulbs is selected at random. Diagram 24 shows the result of the survey, such that X represents the number of light bulbs with a lifespan less than six months.

(a) Find the value of p.
(b) Calculate how many light bulbs are still functioning after six months, if 20 light bulbs from the same brand are used.

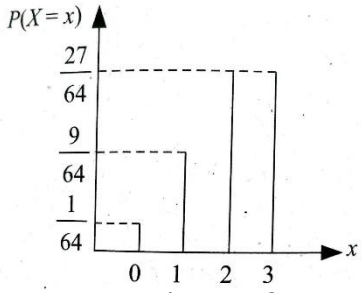


Diagram 24

[4 marks

[(a) $\frac{3}{4}$; (b) 5]

41. SPM 2017 P1 Q24

Diagram 9 shows the graph of binomial distribution $X \sim B(3, p)$.

(a) Express $P(X = 0) + P(X > 2)$ in terms of a and b

(b) Find the value of p.

[3 marks]

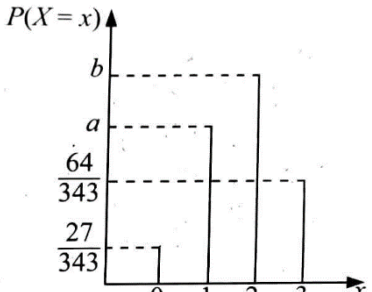


Diagram 9

[(a) $1 - a - b$; (b) $p = \frac{4}{7}$]

42. SPM 2018 P1 Q24

A voluntary body organizes a first aid course 4 times per month, every Saturday from March until September. [Assume there are four Saturdays in every month]

Sueraya intends to join the course but she might need to spare a Saturday per month to accompany her mother to the hospital. The probability that Sueraya will attend the course each Saturday is 0.8. Sueraya will be given a certificate of monthly attendance if she can attend the course at least 3 times a month.

(a) Find the probability that Sueraya will be given the certificate of monthly attendance.

(b) Sueraya will qualify to sit for the first aid test if she obtains more than 5 certificates of monthly attendance. Find the probability that Sueraya qualifies to take the first aid test.

[4 marks]

[(a) 0.8192 ; (b) 0.6301]

43. SPM 2005 P1Q 25

The masses of students in a school have a normal distribution with a mean of 54 and a standard deviation of 12 kg. Find

(a) the mass of the student which gives a standard score of 0.5.

(b) the percentage of students with a mass of greater than 48 kg.

[4 marks]

[(a) 60 kg ; (b) 69.15%]

44. SPM 2006 P1 Q25

Diagram 10 shows a standard normal distribution graph.

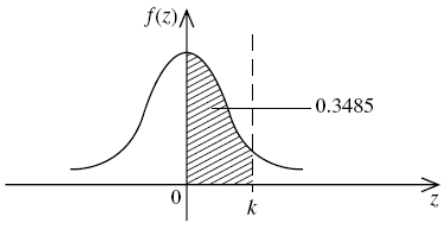


Diagram 10

The probability represented by the area of the shaded region is 0.3485.

(a) Find the value of k.

X is a continuous random variable which is normally distributed with a mean of 79 and a standard deviation of 3. Find the value of X when the z-score is k.

[4 marks]

[(a) 1.03 ; (b) 82.09]

45. SPM 2008 P1Q25

The masses of a group of students in a school have a normal distribution with a mean of 40 kg and a standard deviation of 5 kg.

Calculate the probability that a student chosen at random from this group has a mass of

(a) more than 45 kg.

(b) between 35 kg and 47.8 kg.

[4 marks]

[(a) 0.1587] ; (b) 0.78196]

46. SPM 2009 P1 Q25

The masses of apples in a stall have a normal distribution with a mean of 200 g and a standard deviation of 30 g.

(a) Find the mass, in g, of an apple whose z-score is 0.5.

(b) If an apple is chosen at random, find the probability that the apple has a mass of at least 194 g.

[4 marks]

[(a) 215 ; (b) 0.57926]

47. SPM 2014 P1 Q 25

Diagram 25 shows the standard normal distribution graph with mean, μ, and standard deviation, σ.

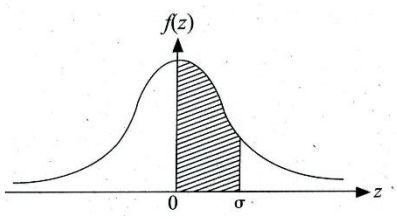


Diagram 25

(a) State

(i) the value of μ,

(ii) the value of σ

(b) Find the area of the shaded region.

[4 marks]

[(a) 0, 1 ; (b) 0.3413]

48. SPM 2015 P1 Q25

The random variable X has a normal distribution with a mean of 25 and a standard deviation of σ. It is given that the z-score is 2 when $X = 26.4$. Find

(a) the value of σ

(b) the value of k such that $P(X > k) = 0.3085$

[4 marks]

[(a) 0.7 ; (b) 25.35]

49. SPM 2017 P1 Q25

Diagram 10 shows a standard normal distribution graph.
The probability represented by the area of the shaded region is 0.2881.

(a) Find the value of h.

(b) X is a continuous random variable which is normally distributed with a mean, μ and a variance of 16.
Find the value of μ if the z-score of $X = 58.8$ is h.

[4 marks]

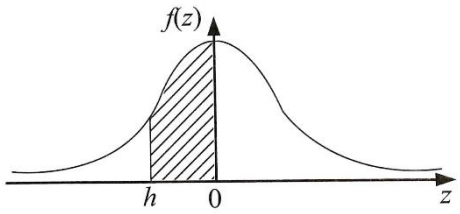


Diagram 10

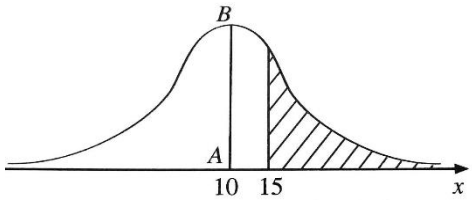
[(a) -0.8 ; (b) 62]

50. SPM 2017 P1 Q25

Diagram 1 shows a probability distribution graph for a random variable X, $X \sim N(\mu, \sigma^2)$.
It is given that AB is the axis of symmetry of the graph.

(a) State the value of μ.

(b) If the area of shaded region is 0.38, state the value of $P(5 \leq X \leq 15)$.



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[2 marks]

[(a) 10; (b) 0.24]

51. SPM 2019 K1 Q25

Diagram 11 shows the normal distribution graph of the time for a school bus to arrive at a school.

(a) Find the standard deviation.

(b) It is given that the mean time for the bus to arrive at the school is 7: 15 a.m. Students are considered late if they arrived after 7:20 a.m.

Lea takes that bus to the school.
Calculate the probability that Lea will be late. Give your answer correct to three significant figures.

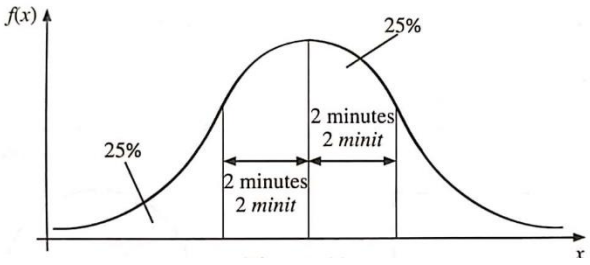


Diagram 11

[4 marks

[(a) 2.967 ; (b) 0.0460]

PAPER 2

52. SPM 2010 P2 Q11

(a) A random variable, X, has a binomial distribution with 10 trials where the probability of success in each trial is p. The mean number of success is 4. Calculate

(i) the value of p, [0.4]

(ii) $P(X \leq 2)$. [0.1673]

[5 marks]

(b) The diameter of limes from a farm has a normal distribution with a mean of 3.2 cm and a standard deviation of 1.5 cm.
Calculate
(i) the probability that a lime chosen at random from this farm has a diameter more than 3.9 cm, [0.32025]
(ii) the value of d if 33% of the limes have diameters less than d cm. [2,54]

[5 marks]

53. SPM 2012 P2 B11

(a) In a survey carried out in a particular district, it is found that three out of five families own a national car. If 10 families are chosen at random from the district, calculate the probability that at least 8 family own a national car. [0.1673]

[4 marks]

(b) In a school, 300 students sat for a test. The marks obtained follow a normal distribution with a mean of 56 and a standard deviation of 8.
(i) Find the number of students who pass the test if the passing mark is 40. [0.9773]
(ii) If 12% of the students pass the test with grade A, find the minimum mark to obtain grade A. [66]

[6 marks]

54. SPM 2013 P2 B11

(a) It is found that 70% of the students from a certain class obtained grade A in Geography in SPM trial examination.
If 10 students from the class are selected at random, find the probability that
(i) exactly 8 students obtained grade A, [0.2335]
(ii) not more than 8 students obtained grade A. [0.8507]

[4 marks]

(b) The Cumulative Grade Point Average (CGPA) of the final year students in a university follows a normal distribution with a mean of 2.6 and a standard deviation of 0.25.

(i) If one student is randomly selected, find the probability that the CGPA of the student is more than 3.0. [0.0548]

(ii) A degree will be given to the final year students who obtained CGPA more than k. If 89.5% of the students manage to get a degree, find the value of k. [2.287]

[6 marks]

55. SPM 2014 P2 B7

A Survey is carried out about a scout in a school.

(a) It is found that the mean of the number of scouts is 315, the variance is 126 and the probability that a student participate in scout is p.

(i) Find the value of p. [0.6]

(ii) If 8 students from the school are chosen at random, find the probability that more than 5 students participate in scout. [0.3154]

[5 marks]

(b) The mass of the scout members in the school follows a normal distribution with a mean of 48 kg and a standard deviation of 5.8 kg.

Find

(i) the probability that a member chosen at random from the group has a mass less than 45 kg. [0.3026]

(ii) the value of m, if 25% of the scout members have mass more than m kg. [51.91]

[5 marks]

56. SPM 2015 P2 B10

(a) The probability of a student cycles to school is p. A sample of 5 students is selected at random.

(i) If the probability of all the 5 students cycle to school is 0.16807, find the value of p. [0.7]

(ii) Find the probability that more than 3 students cycle to school [0.52822]

[5 marks]

(b) Diagram 10 shows a standard normal distribution graph representing the volume of chili sauce in bottles produced by a factory.
It is given that the mean is 950 cm^3 and the variance is 256 cm^6.
If the percentage of the volume more than V is 30.5%, find
(i) the value of V, [958.16]
(ii) the probability that the volume between 930 cm^3 and 960 cm^3. [0.6284]

[5 marks]

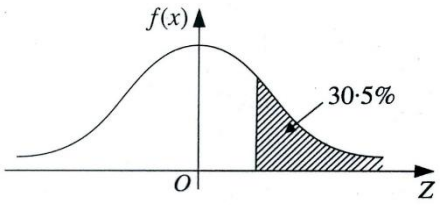


Diagram 10

57. SPM 2016 P2 B9

(a) It is found that 20% of the students from Kampung Aman walk to school. If 8 students from Kampung Aman are chosen at random, find the probability that exactly 3 of them walk to school. [0.1468]

[2 marks]

(b) The mass of pineapples harvested from a farm follows a normal distribution with a mean of 8.22 kg and a standard deviation of m kg. It is given that 15.87% of the pineapples have a mass more than 2.5 kg.
(i) Calculate the value of m. [0.5]
(ii) Given the number of pineapples harvested from the farm is 1 320, find the number of pineapples that have the mass between, 1.0 kg and 2.5 kg. [1080]

[8 marks]

58. SPM 2017 P2 B9

(a) The mass of honeydews produced in a plantation is normally distributed with a mean of 0.8 kg and a standard deviation of 0.25 kg. The honeydews are being classified into three grades A, B and C according to their masses:

Grade A > Grade B > Grade C

(i) The minimum mass of a grade A honeydew is 1.2 kg. If a honeydew is picked at random from the plantation, find the probability that the honeydew is of grade A. [0.0548]

(ii) Find the minimum mass, in kg, of grade B honeydew if 20% of the honeydews are of grade C. [0.5895 kg]

[5 marks]

(b) At the Shoot the Duck game booth at an amusement park, the probability of winning is 25%. Jason bought tickets to play n games. The probability for Jason to win once is 10 times the probability of losing all games.
(i) Find the value of n, [30]
(ii) Calculate the standard deviation of the number of wins. [2.732]

[5 marks]

59. SPM 2018 P2 Q9

A study shows that the credit card balance of the customers is normally distributed as shown in Diagram 6.

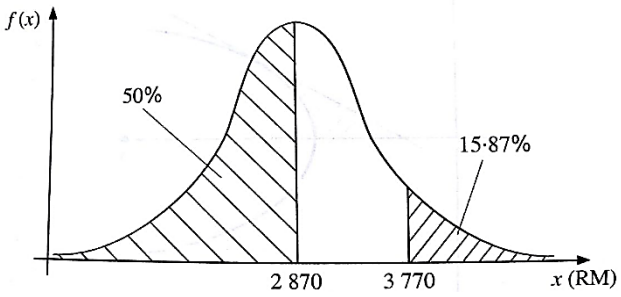


Diagram 6

(a) (i) Find the standard deviation. [900]
(ii) If 30 customers are chosen at random, find the number of customers who have the credit card balance between RM1800 and RM3000. [13]

[7 marks]

(b) It is found that 25% of the customers have the credit card balance less than RMy. Find the value of y. [RM2263.40]

[3 marks]

60. SPM 2019 P2 Q8

Diagram 4 shows a dart's target board at a dart game booth in a funfair.

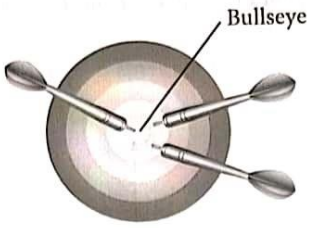


Diagram 4

The booth offers 3 darts per game. The customers have to pay RM5 to play a game. A toy bear will be given to customers who are able to hit the bullseye for the three darts throws in a game. Bob is a dart player. By average, he hits the bullseye 7 times out of 10 darts thrown.

(a) Bob would play the game if he had at least 90% chance to win at least one toy bear by spending RM30. By mathematical calculation, suggest to Bob whether he should play the game or otherwise. [0.9196]

[7 marks]

(b) What is the minimum number of games that Bob needed so that he can get toy bears? [12]

[3 marks]

TRIGONOMETRIC FUNCTIONS

- ONE PAGE NOTE (OPN)
- WORKSHEET

Puan Norfadzilah Lee

NOTA ONE PAGE
" TRIGONOMETRY FUNCTIONS

For any (90 − θ, θ)

- $\sin(90-\theta) = \cos\theta$
- $\cos(90-\theta) = \sin\theta$
- $\tan(90-\theta) = \cot\theta$
- $\operatorname{cosec}(90-\theta) = \sec\theta$
- $\sec(90-\theta) = \operatorname{cosec}\theta$
- $\cot(90-\theta) = \tan\theta$

BASIC IDENTITY

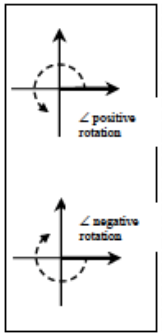
- $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$
  $\Rightarrow \sin^2 x = 1 - \cos^2 x$
- $\sec^2 x = 1 + \tan^2 x \Rightarrow \tan^2 x = \sec^2 x - 1$
- $\operatorname{cosec}^2 x = 1 + \cot^2 x \Rightarrow \cot^2 x = \operatorname{cosec}^2 x - 1$

- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

ADDING FORMULAE

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

ROTATION ANGLE / RELATION BETWEEN ANGLE / 3 BASIC TRIGONOMETRY RATIOS / PYTHAGORAS THEOREM



QUADRANT II ~ obtuse (90° < θ < 180°)
- sin θ → +
- cos θ → −
- tan θ → −

QUADRANT I ~ acute (0° < θ < 90°)
- sin θ → +
- cos θ → +
- tan θ → +

QUADRANT III ~ reflex (180° < θ < 270°)
- sin θ → −
- cos θ → −
- tan θ → +

QUADRANT IV ~ reflex (270° < θ < 360°)
- sin θ → −
- cos θ → +
- tan θ → −

sin: + (I, II) / − (III, IV)

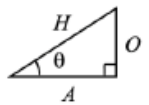
cos: + (I, IV) / − (II, III)

tan: + (I, III) / − (II, IV)

I ⟷ II : 180 − I / 180 − II

I ⟷ III : 180 + I / III − 180

I ⟷ IV : 360 − I / 360 − IV



$H = \sqrt{O^2 + A^2}$

$O = \sqrt{H^2 - A^2}$

$A = \sqrt{H^2 - O^2}$

$\sin\theta = \frac{O}{H}$

$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

	3	4	5		5	12	13	
	6	8	10		7	24	25	
	9	12	15		8	15	17	

Basic Trigonometric Ratio 2

$\sec x = \frac{1}{\cos x} \Rightarrow \cos x = \frac{1}{\sec x}$

$\operatorname{cosec} x = \frac{1}{\sin x} \Rightarrow \sin x = \frac{1}{\operatorname{cosec} x}$

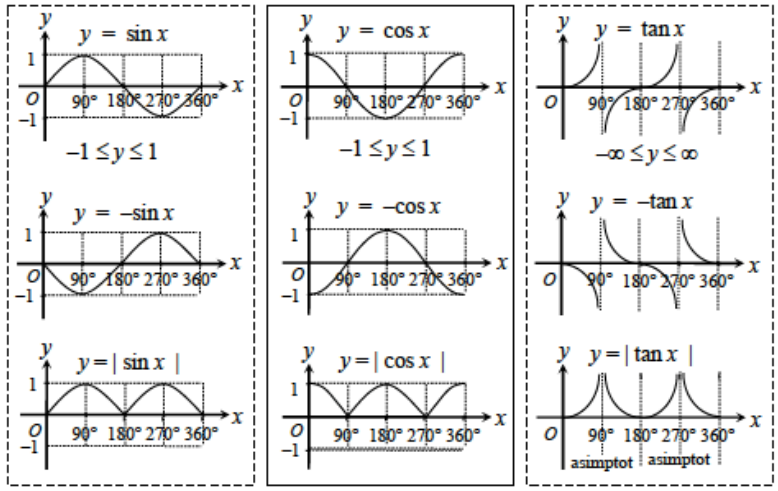
$\cot x = \frac{1}{\tan x} \Rightarrow \tan x = \frac{1}{\cot x}$

$\tan x = \frac{\sin x}{\cos x} \Rightarrow \cot x = \frac{\cos x}{\sin x}$

NEGATIVE ANGLE

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$

GRAPH → $y = a\sin bx$ / $y = a\cos bx$ / $y = a\tan bx$; with $a = 1$, $b = 1$



DOUBLE ANGLES / HALF ANGLES

- $\sin 2A = 2\sin A\cos A \Rightarrow \sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$
- $\cos 2A = \cos^2 A - \sin^2 A \Rightarrow \cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$
  $= 2\cos^2 A - 1 \qquad = 2\cos^2\frac{A}{2} - 1$
  $= 1 - 2\sin^2 A \qquad = 1 - 2\sin^2\frac{A}{2}$
- $\tan 2A = \frac{2\tan A}{1 - \tan^2 A} \Rightarrow \cot 2A = \frac{1 - \tan^2 A}{2\tan A}$

- For graph $y = a\sin bx$ / $y = a\cos bx$ / $y = a\tan bx$
  → amplitude = $|a|$
  $[b = \frac{360}{cycle}]$
- For graph $y = a\sin bx$ / $y = a\cos bx$ → cycle = $\frac{360}{b}$
- For graph $y = a\tan bx$ → cycle = $\frac{180}{b}$ → $\frac{180}{cycle}$

*cycle = angle for repeated graph shape

SPECIAL ANGLES

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

- $\sin x° = 0 \Rightarrow x = 0, 180, 360$
- $\sin x° = 1 \Rightarrow x = 90$
- $\sin x° = -1 \Rightarrow x = 270$
- $\cos x° = 0 \Rightarrow x = 90, 270$
- $\cos x° = 1 \Rightarrow x = 0, 360$
- $\cos x° = -1 \Rightarrow x = 180$
- $\tan x° = 0 \Rightarrow x = 0, 180, 360$
- $\tan x° = \infty \Rightarrow x = 90, 270$

WORKSHEET
TOPIC 6 : TRIGONOMETRIC FUNCTIONS

1 Given that $\tan\theta = t$, $0° < \theta < 90°$, express in terms of t :

(a) $\cot\theta$, (*Ans* : $\frac{1}{t}$)

(b) $\sin(90 - \theta)$. (*Ans* : $\frac{1}{\sqrt{1+t^2}}$)

[3 marks] [***2003, No.20***]

Answer :

(a) (b)

2 Given that $\sin\theta = p$, where p is a constant and $90° \le \theta \le 180°$. Find in terms of p

(a) $\text{cosec}\,\theta$, (*Ans* : $\frac{1}{p}$)

(b) $\sin 2\theta$. (*Ans* : $-2p\sqrt{1-p^2}$

[3 marks] [***2008, No.17***]

Answer :

(a) (b)

3 It is given that $\sin A = \frac{5}{13}$ and $\cos B = \frac{4}{5}$, where A is an obtuse angle and B is an acute angle. Find

(a) $\tan A$, (*Ans* : $-\frac{5}{12}$)

(b) $\cos(A - B)$. (*Ans* : $-\frac{33}{65}$)

[3 marks] [***2009, No.17***]

Answer :

(a) (b)

4 Given $\cos\theta = p$, find $\tan^2\theta$. (*Ans* : $\frac{1-p^2}{p^2}$)

[2 marks] [***2010, No.18***]

Answer :

5 It is given that $\tan A = \frac{3}{4}$ and $\tan B = \frac{7}{24}$, where A is an acute angle and B is a reflex angle. Find

(a) cot A, (*Ans* : $\frac{4}{3}$)

(b) sin (A + B). (*Ans* : $-\frac{4}{5}$)

[3 marks] **[*2011, No.15*]**

Answer :

(a) (b)

6 Given $\cos\theta = -\frac{3}{5}$ and $0° < \theta < 180°$, find the value of $\tan(\theta + 45°)$. (*Ans* : $-\frac{1}{7}$)

[3 marks] [***2015, No.13***]

Answer :

7 Solve the equation $6\sec^2 A - 13\tan A = 0$ for $0° < A < 360°$.
(*Ans* : 33.69°, 213.69° ; 56.31°, 236.31°)

[4 marks] [***2003, No.21***]

Answer :

8 Solve the equation $\cos^2 x - \sin^2 x = \sin x$ for $0° \le x \le 360$. (*Ans* : 30°, 150°, 270°)

[4 marks] [***2004, No.18***]

Answer :

9 Solve the equation $3\cos 2x = 8\sin x - 5$ for $0° \le x \le 360$. (*Ans* : 41.81°, 138.19°)

[4 marks] [***2005, No.17***]

Answer :

10 Solve the equation $15\sin^2 x = \sin x + 4\sin 30°$ for $0° \le x \le 360$.
(*Ans* : 23.58°, 156.42° ; 199.47, 340.53°)

[4 marks] [***2006, No.15***]

Answer :

11 Solve the equation $\cot x + 2\cos x = 0$ for for $0° \le x \le 360$. (*Ans* : 90°, 270° ; 210°, 330°)

[4 marks] [***2007, No.17***]

Answer :

12 Solve the equation $3 \sin x \cos x - \cos x = 0$ for $0° \le x \le 360°$.
(*Ans* : 90°, 270° ; 19.47°, 160.53°) [3 marks] [***2009, No.16***]

Answer :

13 Solve the equation $\sin 2\theta = \cos \theta$ for $0° \le \theta \le 360°$. (*Ans* : 90°, 270° ; 30°, 150°)
[4 marks] **[*2011, No.14*]**

Answer :

14 Solve the equation $\tan^2\theta - 3\tan \theta + 2 = 0$ for $0° \le \theta \le 360°$.
(*Ans* : 45°, 225° ; 63.43°, 243.43°)
[3 marks] **[*2012, No.17*]**

Answer :

15 Solve the equation $\sin 2x + \cos x = 0$ for $0° \le x \le 360°$.
(*Ans* : 90, 270 ; 210, 330)
[4 marks] [***2014, No.14***]

Answer :

16 (a) Sketch the graph of $y = \cos 2x$ for $0 \le x \le 180$. [3 marks]

(b) Hence, by drawing a suitable straight line on the same axes, find the number of solutions which satisfy the equation $2 \sin^2 x = 2 - \frac{x}{180}$ for $0° \le x \le 180°$.

(*Ans* : $y = \frac{x}{180} - 1$, 2) [3 marks]

[***2004, No.3***]

Answer :

17 (a) Sketch the graph $y = -2 \cos x$ for $0° \le x \le 2\pi$. [4 marks]

(b) Hence, using the same axes, sketch a suitable graph to find the number of solutions to the equation $\frac{\pi}{x} + 2\cos x = 0$ for $0° \le x \le 2\pi$. State the number of solutions.

(*Ans* : $y = \frac{\pi}{x}$, 2) [3 marks]

[***2006, No.4***]

Answer :

18 (a) Sketch the graph of $y = |3\cos 2x|$ for $0° \le x \le 2\pi$. [4 marks]

(b) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $2 - |3\cos 2x| = \frac{x}{\pi}$ for $0° \le x \le 2\pi$. State the number of solutions.

(*Ans* : $y = 2 - \frac{x}{\pi}$; 8) [3 marks]

[***2007, No.3***]

Answer :

19 (a) Sketch the graph of $y = \frac{3}{2}\cos 2x$ for $0 \le x \le \frac{3}{2}\pi$. [3 marks]

(b) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $\frac{4}{3\pi}x - \cos 2x = \frac{3}{2}$ for $0 \le x \le \frac{3}{2}\pi$. State the number of solutions.

(*Ans* : $y = \frac{2x}{\pi} - \frac{9}{4}$, 3) [3 marks]

[***2009, No.4***]

Answer :

20 (a) Sketch the graph of $y = 1 + 3\cos x$ for $0 \le x \le 2\pi$. [4 marks]

(b) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $6\pi \cos x = 4\pi - 3x$ for $0 \leq x \leq 2\pi$. State the number of solutions.

(*Ans* : $y = 3 - \frac{3x}{2\pi}$, 2) [3 marks]

[***2010, No.2***]

Answer :

21 (a) Sketch the graph of $y = -3\sin\frac{3}{2}x$ for $0 \leq x \leq 2\pi$. [4 marks]

(b) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $\frac{\pi}{x} + 3\sin\frac{3}{2}x = 0$ for $0 \leq x \leq 2\pi$. State the number of solutions.

(*Ans* : $y = \frac{\pi}{x}$, 2) [3 marks]

[***2011, No.6***]

Answer :

22 (a) Sketch the graph $y = 1 + \tan 2x$ for $0 \leq x \leq \pi$. [3 marks]

(b) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $x + \pi \tan 2x = 0$ for $0 \leq x \leq \pi$.

State the number of solutions. (*Ans* : $y = 1 - \frac{x}{\pi}$, 3) [3 marks]

[***2014, No.2***]

Answer :

23 (a) Prove that $\text{cosec}^2 x - 2\sin^2 x - \cot^2 x = \cos 2x$. [2 marks]

(b) (i) Sketch the graph $y = \cos 2x$ for $0° \leq x \leq 2\pi$.

(ii) Hence, using the same axes, draw a suitable straight line to find the number of solutions to the equation $3\,(\text{cosec}^2 x - 2\sin^2 x - \cot^2 x) = \frac{x}{\pi} - 1$ for $0° \le x \le 2\pi$. State the number of solutions. (*Ans* : $y = \frac{x}{3\pi} - \frac{1}{3}$, 4)

[6 marks]

[***2005, No.8***]

Answer :

24 (a) Prove that $\frac{2\tan x}{2 - \sec^2 x} = \tan 2x$. [2 marks]

(b) (i) Sketch the graph of $y = -\tan 2x$ for $0 \le x \le \pi$.

(ii) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $\frac{3x}{\pi} + \frac{2\tan x}{2 - \sec^2 x} = 0$ for $0 \le x \le \pi$. (*Ans* : $y = \frac{3x}{\pi}$, 3)

[6 marks]

[***2008, No.4***]

Answer :

25 (a) Prove that $\frac{2}{\cos 2x + 1} = \sec^2 x$. [2 marks]

(b) (i) Sketch the graph of $y = \cos 2x + 1$ for $0 \le x \le 2\pi$. [3 marks]

(ii) Hence, use the same axes, sketch a suitable straight line to find the number of solutions for the equation $\frac{2}{\sec^2 x} = \frac{x}{4\pi} + 1$ for $0 \le x \le 2\pi$. State the number of solutions.

(*Ans* : $y = \frac{x}{4\pi} + 1$; 4) [3 marks]

[***2012, No.6***]

Answer :

26 (a) Prove $\frac{\sin 2x}{\tan^2 x + 2\cos^2 x - \sec^2 x} = \tan 2x$. [2 marks]

(b) Skecth the graph of $y = |\tan 2x|$ for $0 \le x \le 2\pi$. [3 marks]

(c) Hence, using the same axes, sketch a suitable straight line to find the number of solution for the equation $\left|\frac{\sin 2x}{\tan^2 x + 2\cos^2 x - \sec^2 x}\right| + \frac{x}{2\pi} = 1$ for $0 \le x \le 2\pi$.

(*Ans* : $y = 1 - \frac{x}{2\pi}$; 8) [3 marks]

[***2016, No.4***]

Answer :

27 (a) Prove that $\tan x \sin 2x = 1 - \cos 2x$. [2 marks]

(b) Hence, solve the equation $\tan x \sin 2x = \frac{1}{4}$ for $0° \le x \le 360°$.

(*Ans* : 20.705°, 159.295°, 200.705°, 339.295°) [4 marks]

[***2013, No.4***]

Answer :

28 (a) (i) Prove that

$2\cos(x+45°)\cos(x-45°) = \cos 2x.$ [3 marks]

(ii) Hence, solve the equation :

$$2\cos(x+45°)\cos(x-45°) = \frac{1}{2} \quad \text{for } 0° \le x \le 360°.$$

(*Ans* : 30, 150, 210, 330) [4 marks]

(b) Sketch the graph of $y = \cos 2x$ for $0° \le x \le 360°$. [3 marks] [***2015, No.***]

Answer :

29 (a) Prove $\sin\left(3x+\frac{\pi}{6}\right) - \sin\left(3x-\frac{\pi}{6}\right) = \cos 3x.$ [3 marks]

(b) Hence,

(i) solve the equation $\sin\left(\frac{3x}{2}+\frac{\pi}{6}\right) - \sin\left(\frac{3x}{2}-\frac{\pi}{6}\right) = \frac{1}{2}$ for $0 \le x \le 2\pi$ and give your answer in the simplest form in terms of π rad, (*Ans* : $\frac{2}{9}\pi$, $\frac{10}{9}\pi$, $\frac{14}{9}\pi$)

(ii) sketch the graph of $y = \sin\left(3x+\frac{\pi}{6}\right) - \sin\left(3x-\frac{\pi}{6}\right) - \frac{1}{2}$ for $0 \le x \le \pi$.

[7 marks]
[***2018 No.7***]

Answer :

30 (a) Prove that $\tan x + \cot x = 2 \operatorname{cosec} 2x$. [4 marks]

(b) (i) Sketch the graph of $y = 2\cos\frac{3}{2}x$ for $0° \le x \le 2\pi$.

(ii) Find the equation of a suitable straight line for solving the equation $\cos\frac{3}{2}x = \frac{3}{4\pi}x - 1$. Hence, using the the same axes, sketch the straight line and state the number of solutions to the equation $\cos\frac{3}{2}x = \frac{3}{4\pi}x - 1$ for $0° \le x \le 2\pi$. (*Ans* : $y = \frac{3}{2\pi}x - 2$, 3)

[6 marks][***2003, No.8***]

Answer :

31 (a) Prove that $2\tan\theta \cos^2\theta = \sin 2\theta$. [2 marks]

(b) Hence, solve the equation $4\tan\theta \cos^2\theta = 1$ for $0 \le \theta \le 2\pi$.
(*Ans* : 15°, 75°, 195°, 255°) [2 marks]

(c) (i) Sketch the graph $y = \sin 2\theta$ for $0 \le \theta \le 2\pi$.

(ii) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $4\pi\tan\theta \cos^2\theta = x - 2\pi$ for $0 \le \theta \le 2\pi$. State the number of solutions. (*Ans* : $y = \frac{x}{2\pi} - 1$, 4)

[6 marks]
[***2017, No.10***]

Answer :

32 (a) (i) Prove that $\tan\left(\frac{A}{2}\right) = \frac{1 - cosA}{\sin A}$.

(ii) Hence, without using a calculator, find the value of $\tan 15°$. State your answer in the form $p - \sqrt{q}$, where p and q are constants. (*Ans* : $2 - \sqrt{3}$)

[4 marks]

(b) (i) Sketch the graphi $y = -\frac{3}{2}\sin A$ for $0 \le A \le 2\pi$.

(ii) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $\left(cot\frac{A}{2}\right)(1 - \cos A) = -\frac{A}{2\pi}$ for $0 \le A \le 2\pi$. State the number of solutions. (*Ans* : $y = \frac{3}{4\pi}A$; 3)

[6 marks]

[***2019, No.10***]

Answer :

33 (a) Without using table or calculator, show that :

$$\tan 195° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

[3 markah]

(b) Given that $\tan(X - Y) + 1 = 0$ and $4\tan X + \tan Y = 5$ where X and Y are acute angles. Find the value of X and of Y that satisfy both equations. (*Ans* : $X = 26.57$, $Y = 71.57$) [5 marks]

Answer :

34 (a) Given that $\cos\theta = p$ and $\sin\theta = q$.

(i) Show that $\dfrac{q^2}{1-p} = 1 + p$. [2 marks]

(ii) Find the value of θ where $0° \le \theta \le 180°$ if $3p - 2q^2 = 0$. (*Ans* : 60) [3 marks]

(b) Given that $\tan A = \dfrac{1}{3}$ and $\tan B = \dfrac{1}{2}$ where A and B are acute angles. Without using table or calculator :

(i) find the value of $\tan (A + B)$, (*Ans* : 1) [2 marks]

(ii) hence, find the value of $A + B$. (*Ans* : 45) [1 mark]

Answer :

35 Given that $\dfrac{\cos (A+B)}{\cos (A-B)} = \dfrac{3}{4}$.

(a) Show that $\cos A \cos B = 7 \sin A \sin B$. [3 marks]

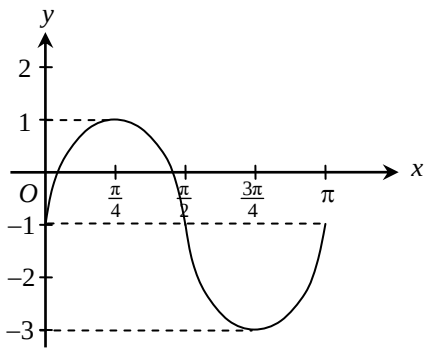
(b) Form a relation between $\tan A$ and $\tan B$. (*Ans* : $\tan A \tan B = \frac{1}{7}$) [2 marks]

(c) If $A + B = 45°$, find the value of $\tan A + \tan B$. (*Ans* : $\frac{6}{7}$) [3 marks]

Answer :

36 (a) Prove that $(\sin x - \cos x)(\tan x + \cot x) = \sec x - \operatorname{cosec} x$. [2 marks]

(b) The diagram shows part of the graph of $y = a\sin bx + c$ for $0 \leq x \leq \pi$.



(i) State the value of a, b and c. [3 marks]

(ii) Find the range of k such that the equation $\sin bx = \frac{k}{a} - \frac{c}{a}$ does not have any real roots. (*Ans* : $k < -3,\ k > 1$) [2 marks]

Answer :

37 (a) Sketch the graph of $y = 3\sin 2x$ for $0° \leq x \leq 2\pi$.

(b) Hence, Find the equation of a suitable straight line for solving the equation $x = \frac{3}{2}\pi\,(1 - \sin 2x)$.
Hence, using the same axes, sketch the straight line and state the number of solutions to the equation $x = \frac{3}{2}\pi\,(1 - \sin 2x)$ for $0° \leq x \leq 2\pi$. (*Ans* : $y = -\frac{2}{\pi}x + 3$, 5)
[6 marks]

Answer :

38 (a) Sketch the graph of $y = -3 \sin 2x$ for $0° \le x \le 2\pi$. [3 marks]

(b) Hence, find the value / values / range of k such that the equation $3 \sin 2x + k = 1$ for $0° \le x \le \pi$ has

(i) no solution, (*Ans* : $k < -2,\ k > 4$)

(ii) only one solution, (*Ans* : $-2,\ 4$)

(iii) only two solutions, (*Ans* : $-2 < k < 1;\ 1 < k < 4$ @ $-2 < k < 4,\ k \neq 1$)

(iv) only three solutions, (*Ans* : 1)

[4 marks]

Answer :

39 (a) Sketch the graph of $y = |-\sin 2x|$ for $0 \le x \le \pi$.

(b) Find the equation of a suitable straight line for solving the equation $2 |\sin 2x| - 1 = 0$. Hence, using the same axes, sketch the straight line and state the number of solutions to the equation $2 |\sin 2x| - 1 = 0$. (*Ans* : $y = \frac{1}{2}$, 4)

[6 marks]

Answer :

40 (a) Sketch the graph of $y = 1 - \sin 2x$ for $0° \le x \le 2\pi$.

(b) Hence, by drawing a suitable straight line on the same axes, find the number of solutions satisfying the equation $x = 2\pi \sin 2x$ for $0 \le x \le 2\pi$. (*Ans* : $y = -\frac{1}{2\pi}x + 1$, 4)

Answer :

41 (a) Sketch the graph of $y = 2|\sin x| - 1$ for $0 \le x \le 2\pi$.

(b) Hence, using the same axes, draw a suitable straight line to find the number of solutions to the equation $2\pi|\sin x| + x = 2\pi$ for $0 \le x \le 2\pi$. State the number of solutions.

(*Ans* : $y = 1 - \frac{1}{\pi}x$, 4)

[6 marks]

Answer :

42 (a) Sketch the graph of $y = -|\cos 2\theta|$ for $0 \leq \theta \leq 2\pi$.

(b) Hence, using the same axes, sketch a suitable graph to find the number of solution to the equation $1 - \left|\frac{1}{\sec 2\theta}\right| = \frac{\theta}{\pi}$ for $0 \leq \theta \leq 2\pi$. State the number of solutions. (*Ans* : $y = \frac{\theta}{\pi} - 1$, 5)

[6 marks]

Answer :

43 (a) Sketch the graph of $y = 2\cos 2x - 1$ for $0 \leq x \leq \pi$.

(b) Hence, using the same axes, sketch a suitable graph to find the number of solution to the equation $\cos 2x = \frac{x}{\pi}$ for $0 \leq x \leq \pi$. State the number of solutions.

(*Ans* : $y = \frac{2}{\pi}x - 1$, 2)

[6 marks]

Answer :

44 (a) Sketch the graph of $y = 2\cos x + 1$ for $0 \le x \le 2\pi$.

(b) Hence, using the same axes, sketch a suitable graph to find the number of solution to the equation $\pi \cos x - \frac{x}{2} = 0$ for $0 \le x \le 2\pi$. State the number of solutions. (*Ans* : $y = \frac{x}{\pi} + 1$, 3)

[6 marks]

Answer :

45 (a) Sketch the graph of $y = 3\tan \frac{1}{2}x$ for $0 \le x \le 2\pi$.

(b) Hence, using the same axes, sketch a suitable graph to find the number of solution to the equation $6\pi \tan \frac{1}{2}x = 2\pi - x$ for $0 \le x \le 2\pi$. State the number of solutions. (*Ans* : $y = 1 - \frac{x}{2\pi}$, 2)

[6 marks]

Answer :

46 (a) Sketch the graph of $y = |\tan x|$ for $0 \leq x \leq 2\pi$.

(b) Hence, using the same axes, sketch a suitable graph to find the number of solution to the equation $2\pi |\tan x| - x = 0$ for $0 \leq x \leq 2\pi$. State the number of solutions. (*Ans* : $y = \frac{x}{2\pi}$, 4)

[6 marks]

Answer :

47 (a) Sketch the graph of $y = 3 \tan x + 2$ for $0 \leq x \leq 2\pi$.

(b) Hence, using the same axes, sketch a suitable graph to find the number of solution to the equation $6 \tan x - 4 = 0$ for $0 \leq x \leq 2\pi$. State the number of solutions. (*Ans* : $y = 4$, 2)

[6 marks]

Answer :

48 (a) Given that $\tan\theta = p$. Show that $\sin 2\theta = \dfrac{2p}{1+p^2}$. [3 marks]

(b) (i) Sketch the graph of $y = \left| \sin x + \dfrac{1}{2} \right|$ for $0 \le x \le 2\pi$. [4 marks]

(ii) Hence, using the same axes, draw a suitable straight line to find the number of solutions to the equation $\left| \sin x + \dfrac{1}{2} \right| = 1 - \dfrac{x}{2\pi}$ for $0 \le x \le 2\pi$.

State the number of solutions. (*Ans* : $y = 1 - \frac{x}{2\pi}$, 5) [3 marks]

Answer :

49 (a) Prove that $\dfrac{\cos(x-y) - \cos(x+y)}{\sin(x+y) + \sin(x-y)} = \tan x$. [3 marks]

(b) (i) Sketch the graph of $y = \left| 1 + 3\cos x \right|$ for $0 \le x \le 2\pi$. [4 marks]

(ii) Hence, using the same axes, draw a suitable straight line to find the number of solutions to the equation $\left| 6\pi \cos x + 2\pi \right| = 4\pi - 3x$ for $0 \le x \le 2\pi$.

State the number of solutions. (*Awp* : $y = 2 - \frac{3x}{2\pi}$, 2) [3 marks]

Answer :

50 (a) Prove that $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$. [2 marks]

(b) Hence,

(i) solve the equation $\frac{2\tan x}{1+\tan^2 x} = \frac{1}{2}$ for for $0 \le x \le 2\pi$ and give your answer in the simplest fraction form in term of π rad, (*Ans* : $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$)

(ii) sketch the graph of $y = \frac{2\tan x}{1+\tan^2 x} - \frac{1}{2}$ for for $0 \le x \le \pi$,

(iii) find the value of k such that the equationn $\frac{2\tan x}{1+\tan^2 x} - \frac{1}{2} = k$ has only three solutions for $0 \le x \le \pi$. (*Ans* : $-\frac{1}{2}$)

[8 marks]

Answer :

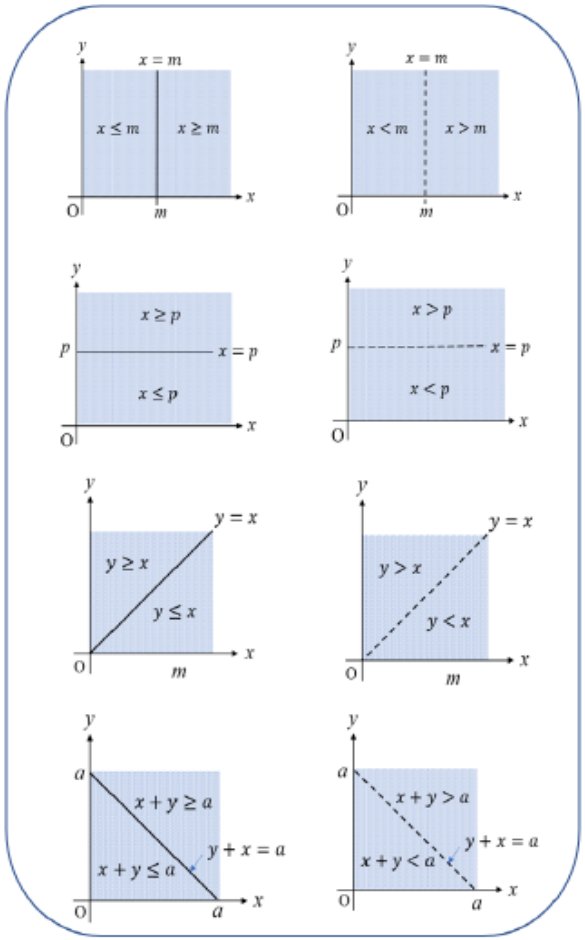
LINEAR PROGRAMMING

- ONE PAGE NOTE (OPN)
- WORKSHEET

Encik Ong Choon Keat

ONE PAGE NOTES
"LINEAR PROGRAMMING "

How to present inequalities graphically?



Keywords

Keywords	Meaning	Determine the symbol < = >			Final Symbol
less than	less than	(<)	=	>	<
more than / exceed	more than	<	=	(>)	>
at least	cannot less than	✗	=	>	≥
at most	cannot more than	<	=	✗	≤
maximum	cannot more than	<	=	✗	≤
minimum	cannot less than	✗	=	>	≥
not less than	cannot less than	✗	=	>	≥
not more than	cannot more than	<	=	✗	≤

No.	Constraints	Inequalities
1.	x is **more than** 3	$x > 3$
2.	y is **less than** 50	$y < 50$
3.	The total of x and y is **at least** 100	$x + y \geq 100$
4.	y is **at most** 2 times of x	$y \leq 2x$
5.	Capital x is a **maximum** of RM500	$x \leq 500$
6.	The **minimum** number of y is 10.	$y \geq 10$
7.	The ratio of x to y is **not less than** 1:3	$\frac{x}{y} \geq \frac{1}{3}$
8.	y exceeds x **not more than** 16.	$y - x \leq 16$

Example:

A school wants to buy two types of tables, M and N to equip a computer lab. The prices for a table M and table N are RM200 and RM100 respectively. The surface area of table M is 1 m^2 while that of table N is 2 m^2. The school intends to buy x units of table M and y units of table N. The purchase of the tables will be based on the following constraints:

I: The total surface area of the tables is not less than 30 m^2.

II: The amount allocated is RM6000.

III: The number of table N is at most twice that of table M.

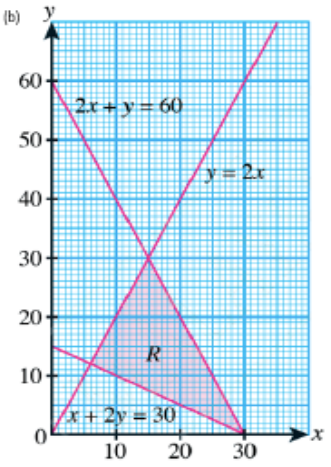
(a) Other than $x \geq 0$ and $y \geq 0$, write three linear inequalities that satisfy all the above constraints.

(b) Using a scale of 2 cm to 10 tables on both the x-axis and the y-axis, construct and label the region R that satisfies all of the above constraints.

Solution:

(a) Constraint I:
$x + 2y \geq 30$
Constraint II:
$200x + 100y \leq 6\,000$
$2x + y \leq 60$
Constraint III:
$y \leq 2x$

(b)



WORKSHEET
TOPIC 7 : LINEAR PROGRAMMING
[2 – 4 questions → 7 – 11 marks]

==========

7.1 Linear Programming Model ~ 3 hours

7.1.1 Form a mathematical model for a situation based on the constraints given and hence represent the model graphically.

[exploratory activities involving optimization need to be carried out]

7.2 Application of Linear Programming ~ 7 hours

7.2.1 Solve problems involving linear programming graphically.

[the terms constraints, scattered region, objective function and optimum value need to be involved]

==========

(A) Linear Programming → Basic

1 Sketch and shade the region that satisfies the given inequalities in each of the following.

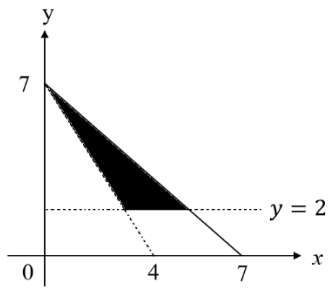
(a) $y \leq x, x + y < 4, 2y \geq x$

(b) $y + x < 5, x + y > 4, 2y \geq x$

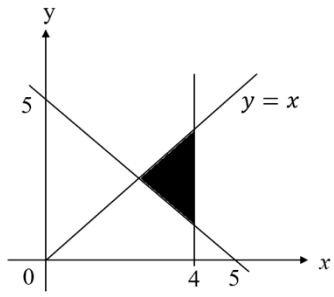
(c) $y \leq x+3, 2y > x, x+y \geq 7$

2 Write the linear inequalities which define the shaded región in each of the following:

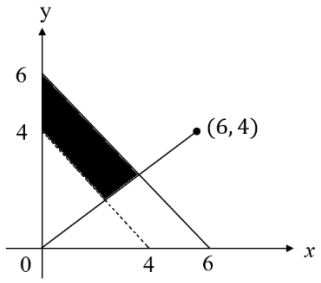
(a)



(b)



(c)



Ans:

(a) $x + y \leq 7, 4y > 28 - 7x, y > 2$

(b) $x \leq 4, y \leq x, x + y \geq 5$

(c) $x + y \leq 6, x + y > 4, 3y \geq 2x$

3 Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints:

The price of a writing book is RM0.80 and the price of a pencil is RM1.20. Siti buys x writing books and y pencils.

I: At least 5 pencils are bought.
II: The total number of writing books and pencils bought are at most 13.
III: The amount spent is not exceed RM10.00.

[Ans: $y \geq 5$, $x + y \leq 13$, $2x + 3y \leq 25$]

Jawapan :

4 Write four inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints:

The price of an orange is RM1.80 and the price of an apple is RM1.20. Fatimah has bought x oranges and y apples.

I: At least 8 oranges are bought.
II: The total number of oranges and apples bought are less than 20.
III: The number of apples bought exceeds two times the number of oranges bought.
IV: The amount spent is at most RM48.00

[*Ans*: $x \geq 8$, $x + y < 20$, $y > 2x$, $3x + 2y \leq 80$]

Jawapan :

5 Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints:

A bakery sells two types of bread, bread M dan bread N. A loaf of bread M costs RM1.90 while a loaf of bread M costs RM1.60. The number of loaves of bread M and bread N sold are x and y respectively.

I: The sales of bread M are at most RM90.
II: The total number of loaves of bread sold is less than 460.
III: The total sales of the two types of bread are at least RM250.

[*Ans*: $x \leq 50$, $x + y < 460$, $19x + 16y \geq 2500$]

Jawapan :

6 Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints:

Ali has RM90 in coins. He has at least RM15 in 10 sen coins and at least RM40 in 50 sen coins. The number of 50 sen coins is more than two times the number of 10 sen coins. The number of 10 sen coins is x and the number of 50 sen coins is y.

[*Ans* : $x \geq 15, y \geq 40, y > 2x$]

Jawapan :

(B) Linear Programming → 10 marks

⇒ ***Part C***

7 *Use graph paper to answer this question*.

Yahya has an allocation of RM 225 to buy x kg of prawns and y kg of fish. The total mass of the commodities is not less than 15 kg. The mass of prawns is at most three times that of fish. The price of 1 kg of prawns is RM 9 and the price of 1 kg of fish is RM 5.

(a) Write down three inequalities, other than $x \geq 0$ and $y \geq 0$, that satisfy all of the above conditions. (*Ans* : $x + y \geq 15$; $y \geq \frac{1}{3}x$; $9x + 5y \leq 225$) [3 marks]

(b) Hence, using a scale of 2 cm to 5 kg for both axes, construct and shade the region R that satisfies all the above conditions. [4 marks]

(c) If Yahya buys 10 kg of fish, what is the maximum amout of money that could remain from his allocation? (*Ans* : 130) [3 marks]

[***2003, No.14***]

Answer :

8 *Use graph paper to answer this question*.

A district education office intends to organize a course on the teaching of Mathematics and Science in English. The course will be attended by x Mathematics participants and y Science participants. The selection of participants is based on the following constraints :

I. The total number of participants is at least 40.

II. The number of Science participants is at most twice that of Mathematics.

III. The maximum alloctation for the course is RM 7200. The expenditure for a Mathematics participants is RM 120 and for a Science participant is RM 80.

(a) Write down three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy the above constraints. (*Ans* : : $x + y \geq 40$; $y \leq 2x$; $3x + 2y \leq 180$) [3 marks]

(b) Hence, by using a scale of 2 cm to 10 participants on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Using the graph from (b), find

(i) the maximum and minimum number of Mathematics participants when the number of Science participants is 10. (*Ans* : $x_{\text{minimum}} = 30$, $x_{\text{maximum}} = 53$)

(ii) the minimum cost to run the course. (*Ans* : 3760)

[4 marks]
[***2004, No.14***]

Answer :

9 *Use graph paper to answer this question.*

An institution offers two computer courses, P and Q. The number of participants for course P is x and for course Q is y. The enrolment of the participants is based on the following constraints :

I. The total number of participants is not more than 100.

II. The number of participants for course Q is not more than 4 times the number of participants for course P.

III. The number of participants for course Q must exceed the number of participants for course P by at least 5.

(a) Write down three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints. (*Ans* : : $x + y \leq 100$; $y \leq 4x$; $y - x \geq 5$) [3 marks]

(b) Hence, by using a scale of 2 cm to 10 participants on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Using the graph from (b), find

(i) the range of the number of participants for course Q if the number of participants for course P is 30. (*Ans* : $35 \leq y \leq 70$)

(ii) the maximum total fees per month that can be collected if the fees per month for course P and Q are RM 50 and RM 60 respectively. (*Ans* : 5800) [4 marks]

[***2005, No.14***]

Answer :

10 *Use graph paper to answer this question.*

A workshop produces two types of rack, P and Q. The production of each type of rack involves two processes, making and painting. The table shows the time taken to make and paint a rack of type P and a rack of type Q.

Rack	Time taken (minutes)	
	Making	Painting
P	60	30
Q	20	40

The workshop produces x racks of type P and y racks of type Q per day. The production of racks per day is based on the following constraints :

I. The maximum total time for making both racks is 720 minutes.

II. The total time for painting both rack is at least 360 minutes.

III. The ratio of the number of racks of type P to the number of racks of type Q is at least 1 : 3.

(a) Write down three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints. (*Ans* : : $3x + y \leq 36$; $3x + 4y \geq 36$, $y \leq 3x$) [3 marks]

(b) Using a scale of 2 cm to 2 racks on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Using the graph from (b), find

(i) the minimum number of racks of type Q if 7 racks of the type P are produced per day. (*Ans* : 4)

(ii) the maximum total profit per day if the profit from one rack of type P is RM 24 and from one rack type Q is RM 32. (*Ans* : 720)

[4 marks]
[***2006, No.13***]

Answer :

11 *Use graph paper to answer this question*.

A factory produces two components, P and Q. In a particular day, the factory produced x pieces of component P and y pieces of component Q. The profit from the sales of a piece of component P is RM15 and a piece of component Q is RM12. The production of the components per day is based on the following constraints :

I. The total number of components produced is at most 500.

II. The number of component P produced is not more than three times the number of component Q.

III. The minimum total profit for both components is RM 4200.

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints.
(*Ans* : : $x + y \leq 500$; $3y \geq x$, $5x + 4x \geq 1400$) [3 marks]

(b) Using a scale of 2 cm to 50 components on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Using the graph from (b), to find

(i) the minimum number of pieces of component Q if the number of pieces of component P produced on a particular day is 100, (*Ans* : 225)

(ii) the maximum total profit per day. (*Ans* : 7125)

[4 marks]
[***2007, No.14***]

Answer :

12 *Use graph paper to answer this question.*

The members of a youth association plan to organise a picnic. The agree to rent x buses and y vans. The rental of a bus is RM800 and the rental of a van is RM300. The rental of the vehicles for the picnic is based on the following constraints :

I. The total number of vehicles to be rented is not more than 8.

II. The number of buses is at most twice the number of vans.

III. The maximum allocation for the rental of the vehicles is RM 4000.

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints. (*Ans* : $x + y \leq 8$; $2y \geq x$; $8x + 3y \leq 40$) [3 marks]

(b) Using a scale of 2 cm to 1 vehicles on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Using the graph from (b), to find

(i) the minimum number of vans rented if 3 buses are rented, (*Ans* : 2)

(ii) the maximum number of member that can be accommodated into the rented vehicles if a bus can accommodate 48 passengers and a van can accommodate 12 passengers. (*Ans* : 216)

[4 marks]

[***2008, No.15***]

Answer :

13 *Use graph paper to answer this question.*

Johan uses x small tiles and y big tiles for decoration of his bathroom. The decoration budget is based on the following constraints :

I. The total number of tiles connot exceed 100 pieces.

II. The number of small tiles connot exceed three times the number of big tiles.

III. The number of big tiles cannot exceed the number of small tiles by more the 30 pieces.

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints. (*Ans* : $x + y \leq 100$; $3y \geq x$; $y - x \leq 30$) [3 marks]

(b) Using a scale of 2 cm to 10 tiles on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Using the graph from (b), to find

(i) the maximum number of small tiles that can be used, (*Ans* : 75)

(ii) the maximum total cost for the tiles if the cost of a small tile is RM1.50 and the cost for a big tile is RM3.00. (*Ans* : 247.5)

[4 marks]
[***2009, No.14***]

Answer :

14 *Use graph paper to answer this question.*

A training centre offer two courses, A and B. The number of participants for course A is x and the number of participants for course B is y. The intake of the participants is based on the following constraints :

I. The maximum number of participants is 80.

II. The number of participants for course B is at least 10.

III. The number of participants for course B is at most $\frac{3}{2}$ times the number of participant for course A.

(a) Write down three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints. (*Ans* : $x + y \leq 80$; $y \geq 10$; $y \leq \frac{3}{2}x$) [3 marks]

(b) Using a scale of 2 cm to 10 participants on both axes, construct and shade the region R which satisfies all of the above constraints. [3 marks]

(c) Using the graph constructed in (b), find

(i) the minimum number of participants for course A, (*Ans* : 7)

(ii) the maximum total fees collected per month if the monthly fees per participant for course A is RM 300 and for course B is RM 400. (*Ans* : 28800)

[4 marks]

[***2010, No.14***]

Answer :

15 *Use graph paper to answer this question.*

A company obtained a painting tender of a government building. The building is to be painted with an undercoat paint and a gloss paint. The number of tins of the undercoat paint is x and the number of tins of the gloss paint is y. The painting works are based on the following constraints :

I. The number of tins of the gloss paint is **not** more than 2 times the number of tins of the undercoat paint,

II. The number of tins of the gloss paint is at least $\frac{1}{4}$ of the number of tins of the undercoat paint,

III. The total time allocated for the painting work is at most 120 hours. For a tin of undercoat paint, the painting time is 3 hours while for a tin of gloss paint, the painting time is 2 hours.

(a) Write down three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints. (*Ans* : $y \leq 2x$, $y \geq \frac{1}{4}x$, $3x + 2y \leq 120$) [3 marks]

(b) Using a scale of 2 cm to 5 tins on both axes, construct and shade the region R which satisfies all of the above constraints. [3 marks]

(c) Using the graph constructed in (b), find

(i) the minimum number of tins of the gloss paint if the number of tins of the undercoat paint is 30. (*Ans* : 8)

(ii) the maximum expenditure for the paint if the price of the undercoat paint is RM 50 per tin and the price of the gloss paint is RM 70 per tin. (*Ans* : 3230)

[4 marks]

[***2011, No.15***]

Answer :

16 *Use graph paper to answer this question.*

A carpenter makes two types of furniture, table and chair. In a week, he mades x tables and y chairs. He has a capital RM 6000. The production of furniture is based on the following constraints :

I. The cost of making a table is RM80 and a chair is RM60.

II. The minimum total number of tables and chairs must be 50.

III. The number of chairs must be at least 80% of the number of tables

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints. (*Ans* : $4x + 3y \leq 300$, $x + y \geq 50$, $y \geq \frac{4}{5}x$) [3 marks]

(b) Using a scale of 2 cm to 10 furniture on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Using the graph constructed in (b), find

(i) the minimum number of chairs made if 24 tables are made, (*Ans* : 26)

(ii) the maximum profit obtained if the profit for a table is RM 40 and the profit for a chair is RM20. (*Ans* : (46, 38) ; 2600)

[4 marks]
[***2012, No.15***]

Answer :

17 *Use the graph provided to answer this question*.

A hardware shop buys x hacksaws and y chisels from a wholesaler. The price of a hacksaw and a chisel are RM40 and RM20 respectively. The purchase of the tools is based on three constraints, two of the constraints are as follows :

I. The total allocation is RM2000.

II. The number of chisels is not more than 4 times the number of hacksaws.

(a) Write two inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy the two constraints.
(*Ans* : $2x + y \leq 100$, $y \leq 4x$) [2 marks]

(b) The third constraint is represented by the shaded region in the graph on next page. Write in words the constrainst. [1 mark]

(c) On the graph, construct and shade the region R which satisfies all the three constraints. [3 marks]

(d) Using the graph in (c), find

(i) the minimum number of chisels if the number of hacksaws is 30, (*Ans* : 10)

(ii) the maximum total number of hacksaws and chisels that can be purchased.
(*Ans* : (17, 66) ; 83)

[4 marks]
[***2016, No.13***]

Answer :

18 *Use graph paper to answer this question.*

An entrepreneur wants to produce two types of toys, P and Q by using a machine. In a day, the machine produces x number of type P toys and y number of type Q toys. The time required to produce a type P toy is 6 minutes and the time required to produce a type Q toy is 5 minutes. The production of the toys is based on the following constraints :

I. The total number of toys produced must be more than 40 units in a day.

II. The machine can operate for only 15 hours a day.

III. The ratio of the number of type P toys to the number of type Q toys is at most 3 : 5.

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$ that satisfy all the above constraints. (*Ans* : $x + y > 40$, $6x + 5y \leq 900$, $y \geq \frac{5}{3}x$) [3 marks]

(b) Using a scale of 2 cm to 20 toys on both axes, construct and shade the region R Which satisfy all the above constraints. [3 marks]

(c) Using the graph constructed in (b), find the range of total sales that can be obtained if the selling price of a type P toy is RM5 and the selling price of a type Q toy is RM3. (*Ans* : $120 < k \leq 625$)

[4 marks]

[***2017, No.14***]

Answer :

19 *Use a graph paper to answer this question.*

A department store sells two types of perfumes, P and Q. The selling Price of perfume P is RM400 per bottle and perfume Q is RM100 per bottle. A promoter, Zie sells x bottles of perfume P and y bottles of perfume Q. She will be given 5% commision of her total sales if she is able to achieve the following targets:

I. The total number of bottles of perfume sold is at least 50.

II. The minimum total sale is RM8000.

(a) Write two inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints.

(*Ans* : $x + y \geq 50$, $4x + y \geq 80$) [2 marks]

(b) Use a scale of 2 cm to 10 bottles on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Use the graph constructed in **(b)** to answer the following questions :

(i) Find the minimum number of bottles of perfume P and perfume Q to be sold if the number of bottles of perfume Q sold is twice the number of bottles of perfume P.

(*Ans* : $P = 17$, $Q = 34$)

(ii) Determine the minimum amount of commision received by Zie if she is able to sell 35 bottles of perfume P. (*Ans* : (35, 15) ; 775) [4 marks]

[***2018, No.15***]

Answer :

20 *Use graph paper to answer this question*.

Siti is assigned to prepare prizes in conjuction with STEM Week. She is required to purchase x set of pens and y set of notebooks. The price for a set of pens is RM3 and the price for a set of notebooks is RM4. The purchase of the prizes is based on the following constraints:

I. The total number of set of pens and set of notebooks must be more than 40.

II. The total allocation is RM400.

III. The number of set of pens exceed the number of set of notebooks by at most 10.

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints.
(*Ans* : : $x + y > 40$; $3x + 4y \leq 400$, $y \geq x - 10$) [3 marks]

(b) Using a scale of 2 cm to 10 sets on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Use the graph constructed from (b) to answer the following questions:

(i) Find the minimum balance of the allocation, if the number of set of pens purchased is equal to the number of set of notebooks. (*Ans* : (57, 57), RM1.00)

(ii) Determine the maximum number of set of pens that can be purchased if Siti spent RM50 for wrapping papers. (*Ans* : 54)

[4 marks]
[***2019, No.14***]

Answer :

21 *Use graph paper to answer this question.*

A school band teacher was given an allocation of RM800 for transportation to participate in the competition. The band team consists of 80 members and 1000 kg of musical instruments. Two types of vans are used for transportation. Van *A* can transport 10 peoples and 500 kg of tools, van *B* can transport 20 peoples and 100 kg of tools. The rent for van *A* and van *B* is RM100 and RM80 respectively. Using x to represent the number of van *A* and y represents the number of van *B* rented.

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints.
(*Jwp* : $x + 2y \geq 8$; $5x + y \geq 10$; $5x + 4y \leq 40$) [3 marks]

(b) Using a scale of 2 cm to 1 van on both axes, construct and shade the region *R* which satisfies all the above constraints. [4 marks]

(c) If the teacher rents 3 van *A*, What is the máximum balance of the allocation ?
(*Jwp* : 260) [3 marks]

Jawapan :

22 *Use graph paper to answer this question.*

A furniture workshop has 2 workers, A and B, doing the job of assembling and then painting desks. The time taken by these workers is as tabulated in the table.

Worker	Time taken (minutes)	
	Assemble	Paint
A	105	60
B	120	30

In a week, worker A can complete x desks, while worker B complete y desks. These two workers work under the following constraints :

I. The total time taken by the two workers does not exceed 110 hours.

II. The minimum total time by the two workers for assembling the desks is 14 hours.

III. The number of desks completed by A must not exceed by 20 that completed by B.

(a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$, which satisfy all the above constraints.
(Ans : : $11x + 10y \leq 440$; $7x + 8y \geq 56$, $y \geq x - 20$) [3 marks]

(b) Using a scale of 2 cm to 5 desks on both axes, construct and shade the region R which satisfies all the above constraints. [3 marks]

(c) Using the graph from (b), find

(i) Find the range of the number of desks which completed by worker B if worker A completed 6 desks for that particular week. (Ans : $2 \leq y \leq 37$)

(ii) The profit for each desk sold is RM12. Assuming all the desks made by A and B for that particular week are sold, find the maximum total profit netted. (Ans : 528)

[4 marks]

Answer :

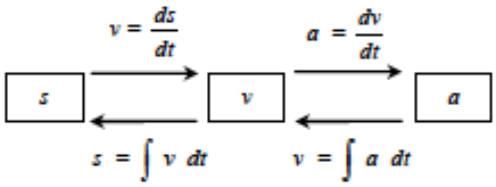
KINEMATICS OF LINEAR MOTION

- ONE PAGE NOTE (OPN)
- WORKSHEET

Puan Lai Hoi Yong

ONE PAGE NOTES

" KINEMATICS OF LINEAR MOTION "



~ s = displacement

~ v = velocity

~ a = acceleration

IMPLIKASI :

- return to / passes O again → $s = 0$.
- to the left of O / negative displacement → $s < 0$;
  to the right of O / positive displacement → $s > 0$.
- P and Q meet each other → $s_P = s_Q$
- stop / stationary / at rest / change direction / reverse / minimum displacement @ maximum displacement → $v = 0$.
- moves to the left / negative velocity → $v < 0$;
  Moves to the right / positive velocity → $v > 0$.
- minimum velocity / maximum velocity, uniform velocity / constant velocity → $a = 0$.
- velocity decrease / deceleration → $a < 0$;
  velocity increase → $a > 0$.
- passes O again (s), initial velocity (v), initial acceleration (a) → $t = 0$.
- distance travelled in the n second / at the n second = $\int_{n-1}^{n} v\ dt = |s_n - s_{n-1}|$
- Total distance travelled by the particle in the first n second ,

  (i) If $v \neq 0$, total distance = $\int_0^n v\ dt = |s_n - s_0|$

  (ii) If $v = 0$ when $t = m$ where $0 \leq m \leq n$, total distance = $\left|\int_0^m v\ dt\right| + \left|\int_m^n v\ dt\right|$

  # first 5 second → $t = 0$ **until** $t = 5$
  # the fifth second → $t = 4$ **until** $t = 5$
- average velocity = $\dfrac{\text{total distance}}{\text{total time}}$

DIFFERENTIATION			
$\frac{d}{dx}(k) = 0$	$\frac{d}{dx}(kx) = k$	$\frac{d}{dx}(kx^n) = (nk)x^{n-1}$	$\frac{d}{dx}\left(\frac{k}{x^n}\right) = \frac{-nk}{x^{n+1}}$
$\frac{d}{dx}(2) = 0$	$\frac{d}{dx}(\frac{x}{8}) = \frac{1}{8}$	$\frac{d}{dx}(-3x^4) = -12x^3$	$\frac{d}{dx}(\frac{3}{x^2}) = \frac{-6}{x^3}$

INTEGRATION		
$\int k\ dx = kx + c$	$\int kx^n\ dx = \frac{kx^{n+1}}{n+1} + c$	$\int \frac{k}{x^n}\ dx = \frac{k}{x^{n-1}[-(n-1)]} + c$
$\int 2\ dx = 2x + c$	$\int 5x^2\ dx = \frac{5x^3}{3} + c$	$\int \frac{5}{x^3}\ dx = \frac{5}{x^2(-2)} + c$

WORKSHEET

TOPIC 8 : KINEMATTICS OF LINEAR MOTION

1 A particle moves along a straight line from a fixed point P. Its velocity, v ms^{-1}, is given by $v = 2t\,(6 - t)$, where t is the time, in second, after leaving the point P. (Assume motion to the right is positive)
Find

(a) the maximum velocity of the particle, (*Ans* : 18) [3 marks]

(b) the distance travelled during the third second, (*Ans* : $17\frac{1}{3}$) [3 marks]

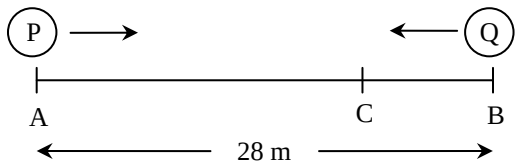
(c) the value of t when the particle passes the point P again, (*Ans* : 9) [2 marks]

(d) the time between leaving P and when the particle reverses its direction of motion.
(*Ans* : 6) [2 marks]

[***2004, No.15***]

Answer :

2 The diagram shows the positions and directions of motion of two objects, P and Q, moving in a straight line and passes through two fixed points, A and B, respectively. Object P passes the fixed point A and object Q passes the fixed point B simultaneously. The distance AB is 28 m.



The velocity of P, v_p ms^{-1}, is given $v_p = 6 + 4t - 2t^2$, where t is the time, in seconds, after it passes A while Q travels with a constant velocity of -2 ms^{-1}. Object P stops instantaneously at the point C. [Assume that the positive direction of motion is towards the right]

Find

(a) the maximum velocity, in ms^{-2}, of P. (*Ans* : 8) [3 marks]

(b) the distance, in m, of C from A, (*Ans* : 18) [4 marks]

(c) the distance, in m, between P and Q when P is at the point C. (*Ans* : 4) [3 marks]

[***2005, No.15***]

Answer :

3 A particle move in a straight line and passes through a fixed point O. It velocity, v ms^{-1}, is given by $v = t^2 - 6t + 5$, where t is the time, in seconds, after leaving O. [Assume motion to the right is positive]

(a) Find

(i) the initial velocity of the particle, (*Ans* : 5)

(ii) the time interval during which the particle moves towards the left, (*Ans* : $1 < t < 5$)

(iii) the time interval during which the acceleration of the particle is positive. (*Ans* : $t > 3$)
[5 marks]

(b) Sketch the velocity-time graph of the motion of the particle for $0 \le t \le 5$. [2 marks]

(c) Calculate the total distance traveled during the first five second after leaving O. (*Ans* : 13)
[3 marks]
[***2006, No.12***]

Answer :

4 A particle moves in a straight line and passes through a fixed point O. It velocity, v ms^{-1}, is given by $v = t^2 - 6t + 8$, where t is the time, in seconds, after passing through O. Find

[Assume motion to the right is positive]

(a) the initial velocity, in ms^{-1}, (*Ans* : 8) [1 mark]

(b) the minimum velocity, in ms^{-1}, (*Ans* : −1) [3 marks]

(c) the range of values of t during which the particle moves to the left, (*Ans* : $2 < t < 4$)[2 marks]

(d) the total distance, in m, travelled by the particle in the first 4 seconds. (*Ans* : 8)[4 marks]
[***2007, No.12***]

Answer :

5 A particle moves along a straight line and passes through a fixed point O. Its velocity, v ms^{-1}, is given by $v = 10 + 3t - t^2$, where t is the time, in seconds, after passing through O. The particle stops instantaneously at a point R. Find

[Assume motion to the right is positive]

(a) the acceleration, in ms^{-2}, of the particle at R, (*Ans* : −7) [3 marks]

(b) the minimum velocity, in ms^{-1}, of the particle, (*Ans* : $12\frac{1}{4}$) [3 marks]

(c) the total distance, in m, travelled by the particle in the first 9 seconds, after passing through O. (*Ans* : $123\frac{1}{6}$) [4 marks]

[***2008, No.12***]

Answer :

6 A particle moves along a straight line and passes through a fixed point O. Its velocity of the particle, v ms^{-1}, is given by $v = 15 + 4t - 3t^2$, where t is the time, in s, after leaving O.

(a) the initial velocity, in ms^{-1}, of the particle, (*Ans* : 15) [1 mark]

(b) the initial acceleration, in ms^{-2}, of the particle, (*Ans* : 4) [2 marks]

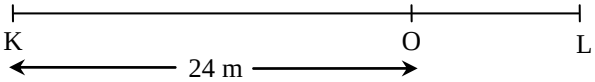
(c) the maximum velocity, in ms^{-1}, of the particle, (*Ans* : $16\frac{1}{3}$) [3 marks]

(d) the distance, in cm, from O when the particle stops instantaneously. (*Ans* : 36) [4 marks]

[***2009, No.15***]

Answer :

7 The diagram shows a straight line KOL.



A particle moves along a straight line and passes through the fixed point O. It velocity. v ms^{-1}, is given by $v = 8 - 4t$, where t is the time, in seconds, after leaving the point O. The particle stop instantaneously by at point L. Find [Assume motion to the right is positive]

(a) the acceleration, in ms^{-2}, of the particle, (*Ans* : –4) [1 mark]

(b) the time, in second, when the particle is at L, (*Ans* : 2) [2 marks]

(c) the velocity, in ms^{-1}, of the particle when it passes through K, (*Ans* : –16) [3 marks]

(d) the total distance, in m, travelled by the particle from O to K passing through L. (*Ans* : 40) [4 marks]

[***2012, No.12***]

Answer :

8 A particle moves along a straight line and passes through a fixed point O. Its velocity, v ms^{-1}, is given by $v = pt^2 + qt$, where p and q are constants and t is the time, in seconds, after passing through O. It is given that the particle stop instantaneously when $t = 4$ s and its acceleration is –2 ms^{-2} when $t = 1$ s. [Assume motion to the right is positive]

Find

(a) the value of p and of q, (*Ans* : $p = 1$, $q = -4$) [5 marks]

(b) the range of values of t when the particle moves to the left, (*Ans* : $0 < t < 4$) [2 marks]

(c) the distance, in m, travelled by the particle during the fourth second. (*Ans* : $1\frac{2}{3}$) [3 marks]

[***2014, No.12***]

Answer

9 A particle moves along a straight line and passes through a fixed point O. Its velocity v ms^{-1}, is given by $v = pt^2 - 6t$, , where p is a constant and t is the time, in seconds, after passing through O . The acceleration of the particle is 18 ms^{-2} when $t = 3s$.

Find

(a) the value of p (*Ans* : 4) [3 marks]

(b) the time interval, in seconds, when the velocity of the particle is decreasing. (*Ans* : $t < \frac{3}{4}$)[2 marks]

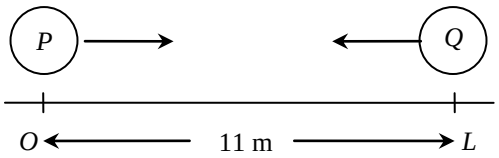
(c) the time, in seconds, when the particle stops instantaneously. , (*Ans* : $\frac{3}{2}$) [2 marks]

(d) the total distance, in m , travelled by the particle in the first 3 seconds. (*Ans* : $\frac{27}{2}$ / $13\frac{1}{2}$)[3 marks]

[***2016, No.14***]

Answer :

10 The diagram shows the initial position and directions of motion of two particles, P and Q, which start moving simultaneously.



The velocity of particle P, v_p ms^{-1}, is given by $v_p = 9t^2 + 10$ and the displacement of particle Q, s_Q m, from point L is given by $s_Q = 3t^3 - t$, where t is the time, in seconds, after particle P passes point O and partivle Q passes point L.

(a) Find the initial velocity , in ms^{-1}, of particle Q, (*Ans* : −1) [2 marks]

(b) Calculate the total distance, in m , travelled by particle Q in the first 2 seconds.

(*Ans* : $22\frac{4}{9}$) [4 marks]

(c) Calculate the distance, in m of the particles from point L when particle P and particle Q meet .

(*Ans* : 2) [4 marks]

[***2017, No.12***]

11 A particle moves in a straight line and pass through a fixed point O, with a velocity of 24ms^{-1}. Its acceleration, $a\ \text{ms}^{-2}$, t s after passing through O is given by $a = 10 - 2t$. The particle stop after k s.

(a) Find

(i) the maximum velocity of the particle, (*Ans* : 49)

(ii) the value of k. (*Ans* : 12)

[6 marks]

(b) Sketch a velocity-time graph for $0 \le t \le k$.

Hence, or otherwise, calculate the total distance travelled during that period. (*Ans* : 432)[4 marks]

[***2003, No.12***]

Answer :

12 A particle moves along a straight line such that its acceleration, $a\ \text{ms}^{-2}$, is given by $a = 10 - 2t$, where t is the time, in seconds, after it passes through a fixed point O. The initial velocity of the particle is $-16\ \text{ms}^{-1}$. Find

(a) the maximum velocity, in ms^{-1}, of the particle, (*Ans* : 9) [4 marks]

(b) the range of values of t during which the particle moves to the right, (*Ans* : $2 < t < 8$)[2 marks]

(c) the total distance, in cm, travelled by the particle in the first 6 seconds. (*Ans* : $41\frac{1}{3}$)[4 marks]

[***2010, No.12***]

Answer :

13 A particle moves along a straight line with an initial velocity of $5\ \text{ms}^{-1}$. Its acceleration , $a\ \text{ms}^{-2}$, is given by $a = 9 - 4t$, where t is the time, in seconds, after passing through a fixed point O.

Find

(a) the time, in seconds, when its acceleration is zero, (*Ans* : $\frac{9}{4}$) [1 marks]

(b) the maximum velocity, in m s^{-1} , of particle, (*Ans* : $15\frac{1}{8}$) [3 marks]

(c) the time, in seconds, when the particle stop instantaneously. (*Ans* : 5) [2 marks]

(d) the total distance, in m , travelled by the particle in the first 6 seconds. (*Ans* : $60\frac{1}{3}$) [4 marks]

[***2011, No.12***]

Answer :

14 Two particles P and Q move along a straight line with the condition that the displacement of the particles from a fixed point O is s m. The particle P starts moving from A with $s_P = 10 + 8t - 8t^2$ and at the same time the particle Q starts moving from B with $s_Q = 6t^2 - 9t - 12$with the condition that t is the time, in seconds, after both particles start moving.

(a) Find the distancem in m , between A and B. (*Ans* : 22) [2 marks]

(b) Fnd the time, in seconds, when the two particles meet each other , (*Ans* : 2) [3 marks]

(c) Calculate the total distance, in m , travelled by particle P when both particles meet each other.

(*Ans* : 20) [5 marks]

[***2018, No.12***]

Answer :

15 An object is thrown vertically upward from the top of a building that is 48m above the ground level. The displacement of the object, x m, from the top of the building is given by $x = 8t - 5t^2$ where t is the time in seconds. Find

(a) the maximum distance of the object from ground level, (*Ans* : $51\frac{1}{5}$ m) [5 marks]

(b) the time and the velocity of when the object strikes the ground. (*Ans* : -32 ms^{-1}) [5 marks]

Answer

16 A particle P moves in a straight line and passes through a fixed point O. Its displacement, s m, is given by $s = 11 + 6t^2 - t^3$, where t is the time, in seconds, after leaving O . Find

(a) the maximum distance, in m, of P from O, (*Ans* : 43m) [3 marks]

(b) the maximum velocity, in ms^{-1}, of P (*Ans* : 12 ms^{-1}) [4 marks]

(c) the average speed of P in the first 2 seconds. (*Ans* : 8 ms^{-1}) [3 marks]

Answer

17 A particle moves along a straight line and passes through a fixed point O. Its velocity, v m s^{-1}, is given by $v = pt^2 + qt - 16$, where t is the time, in seconds, after passing through O, p and q are constants. The particle stops momentarily at a point 64 m to the left of O when $t = 4$. Find
[Assume motion to the right is positive.]

(a) the value of p and of q, (*Ans* : $p = 3$, $q = -8$) [5 marks]

(b) the acceleration of the particle when it stops momentarily, (*Ans* : 16) [2 marks]

(c) the total distance traveled in the third second. (*Ans* : 17) [3 marks]

Answer :

18 A particle moves along a straight line and its velocity, v m s^{-1}, is given by $v = t(a - bt)$ where t is the time in second after passing through a fixed point O, while a and b are constants. The particle stops momentarily when $t = 2$ s and its distance at that moment is 4 m to the right of O. Find
(Assume motion to the right is positive)

(a) the value of a and of b. (*Ans* : $a = 6$, $b = 3$) [5 marks]

(b) the value of t when the particle passes the point O again, (*Ans* : 3) [2 marks]

(c) the total distance travelled, in m, in the first three second. (*Ans* : 8) [3 marks]

Answer :

19 A particle moves along a straight line and passes through a fixed point O with velocity 20ms^{-1}. The acceleration of the particle, $a\text{ ms}^{-2}$, is given by $a = 11 - 6t$ where t is the time in seconds after passing through O.

(a) Find the maximum velocity, in ms^{-1}, of the particle, ($Ans: \frac{361}{12}$) [3 marks]

(b) Find the distance, in m, travelled by the particle in the third second, ($Ans: 28\frac{1}{2}$) [3 marks]

(c) Find the time when the distance of the particle from O is maximum. ($Ans:$ 5) [2 marks]

(d) Sketch the velocity-time graph of the motion of the particle for $0 \le t \le 5$. [2 marks]

Answer

WORKSHEET ANSWERS

1) Can be viewed or downloaded using the following link.

bit.ly/MTWORKHEETANSWERF5

or

2) Scan this QR code.

ACKNOWLEDGEMENT

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"Success is not final; failure is not fatal:
It is the courage to continue that counts"